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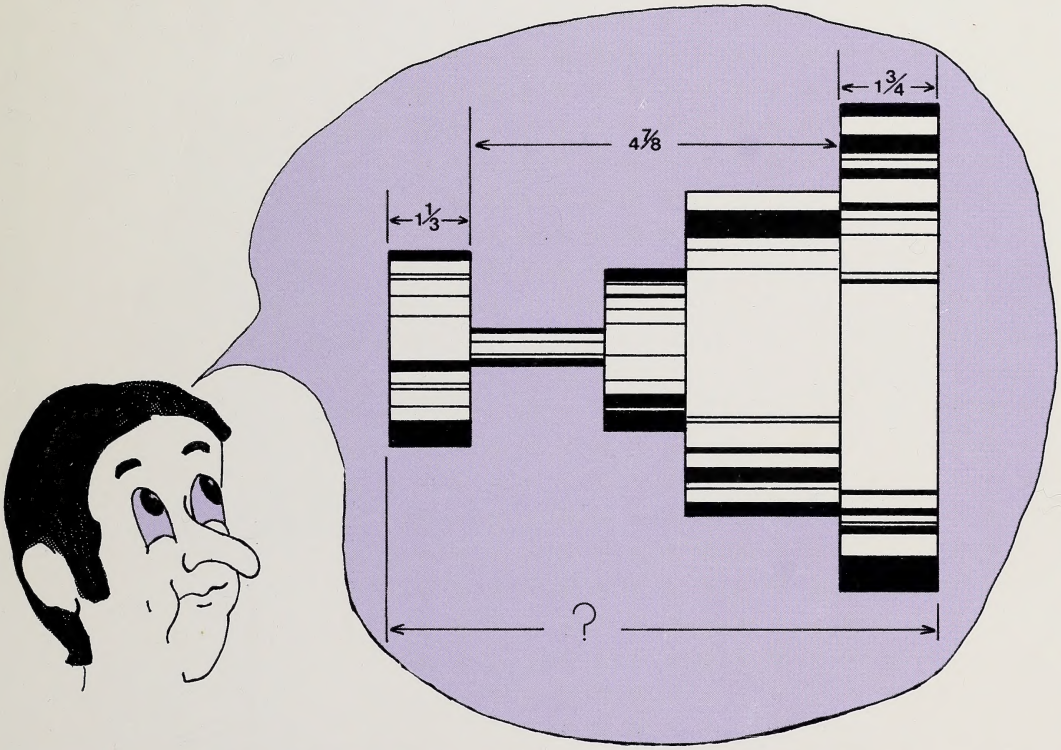
PRACTICAL MATHEMATICS



FRACTIONAL NUMBERS

module 2

ALBERTA CORRESPONDENCE SCHOOL
ALBERTA EDUCATION
BARRHEAD, ALBERTA

Practical Mathematics

Module 2

FRACTIONAL NUMBERS

Practical Mathematics
Student Module
Module 2
Fractional Numbers
Alberta Correspondence School
ISBN No. 0-7741-0290-X

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IMPORTANT!

PLEASE READ THIS INTRODUCTION BEFORE BEGINNING YOUR FIRST LESSON.

Content and Purpose of Course

In its entirety, this Practical Mathematics Series consists of eight modules, each containing from four to seven lessons. The titles of these modules are as follows:

Module 1: Whole Numbers
Module 2: Fractional Numbers
Module 3: Decimal Numbers
Module 4: Ratio and Proportion
Module 5: Measurement
Module 6: Introductory Algebra
Module 7: Intermediate Algebra
Module 8: Trigonometry

Taken individually, these modules should provide a person with instruction and practice in specific areas in which he finds that his mathematical background is weak.

The first five modules, taken as a unit, should equip a person to handle most practical mathematical problems encountered in day-to-day living. The next two modules cover the basics of algebra required for subsequent vocational mathematics courses. The last module covers the basics of trigonometry.

The series, Practical Mathematics, was also developed with the needs of prospective tradesmen in mind. Modules 1-5 inclusive were designed to prepare students mathematically for certain apprenticeship programs. In other words, if a person completes the first five modules of this series, he should be prepared to write the mathematics section of the examination set by the Apprenticeship Board for entrance into the following apprenticeship programs:

Agricultural Mechanic
Appliance Serviceman
Auto Body Mechanic
Baker
Bricklayer
Carpenter
Cook
Floorcovering Mechanic
Gasfitter
Glassworker
Heavy Duty Mechanic
Insulator
Lather
Machinist
Millwright
Motor Mechanic
Painter and Decorator
Partsman
Plasterer
Plumber
Refrigerator Mechanic
Roofer
Sheet Metal Mechanic
Steamfitter
Steel Fabricator
Tile Setter
Welder

If a person completes Modules 1 to 7 inclusive, he should be prepared to write the Apprenticeship Board's mathematics examination for entrance into the apprenticeship programs listed below:

Communication Electrician	Instrument Mechanic
Electrician	Power Electrician
Electrical Mechanic	Radio Technician

There is no prescribed textbook for this course. All the explanatory material and exercises are provided in the lessons.

Self-Correcting Exercises

A number of self-correcting exercises appear in the lessons in this course. You should work through each of these exercises carefully and when you have completed the **entire** exercise, check your answers with those provided at the end of the lesson. Correct any errors that you have made in these exercises. If you do not understand some of the answers, ask your correspondence teacher about them. It is important that you handle these self-correcting exercises properly as they help prepare you for the assignments that follow.

Inventory Tests

Inventory tests appear at the beginning of Modules 2-5 inclusive. These lessons are designed to test the student's grasp of basic skills which should be mastered before additional lessons are attempted. Complete each inventory test and send it in for correction before beginning any other lessons in that module.

Optional Sections

Optional sections occur in some lessons in this course. Students not progressing beyond Module 5 are not required to complete these sections, but may do so if they wish. Students who plan on taking Modules 6, 7 and 8 **must** complete these optional sections.

Dear Student:

As of June 1, 1989 you will not be required to send in any lessons in Practical Mathematics (Modules 1 - 5). At the back of the module you will find a booklet of answers and solutions.

Study the lesson notes very carefully, work on the exercises and then check your work by using this booklet of answers and solutions. If, however, you experience difficulty in doing some questions you may telephone the Alberta Correspondence School at 674-5333. Students who reside in Alberta may call the school free of charge through the local R.I.T.E. number in your area or the Zenith number 22333 (if there is no R.I.T.E. centre).

When you have completed your home study and feel confident that you know the material then you may proceed onto Mathematics 14 (if you require credits) or into Practical Mathematics (Modules 6, 7 and 8) or Basic Algebra and Geometry (if you require an academic mathematics course). However, if you are undecided which route to take then write us a letter or telephone us by using the above number.

We wish you success in this course.

Ronald Billey

Ron Billey
Vice-Principal
Mathematics Science Department

MODULE 2: FRACTIONAL NUMBERS

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Practical Mathematics
Module 2
Lesson 1

Before we go on to discuss fractional numbers, let's make sure you remember how to work with whole numbers. Please complete the following test and send it in for correction as lesson 1 of this module.

INVENTORY TEST — Whole Numbers

Part A

1. Add the following:

(a)
754
836
958
421

(b)
821
569
740
376

(c)
500
258
239
579
801
325
100
932

(d)
600
315
359
747
451
144
659
775

(e) Four thousand nine hundred nine; five hundred fifty-six; nine thousand six hundred; seventy-four; one hundred fifty. Be sure to add these numbers.

(f) One thousand ninety-six; three thousand three hundred twenty-one; nine hundred twenty; nine thousand one hundred eight. Be sure to add these numbers.

2. Subtract the following:

(a) $\begin{array}{r} 840 \\ -611 \\ \hline \end{array}$ (b) $\begin{array}{r} 540 \\ -224 \\ \hline \end{array}$ (c) $\begin{array}{r} 905 \\ -618 \\ \hline \end{array}$ (d) $\begin{array}{r} 8007 \\ -4098 \\ \hline \end{array}$ (e) $\begin{array}{r} 9001 \\ -7806 \\ \hline \end{array}$

(f) Five hundred ninety-seven from one thousand eighty-six

3. Multiply the following whole numbers:

(a) $\begin{array}{r} 93 \\ \times 46 \\ \hline \end{array}$ (b) $\begin{array}{r} 45 \\ \times 91 \\ \hline \end{array}$ (c) $\begin{array}{r} 846 \\ \times 95 \\ \hline \end{array}$ (d) $\begin{array}{r} 805 \\ \times 64 \\ \hline \end{array}$ (e) $\begin{array}{r} 295 \\ \times 751 \\ \hline \end{array}$

(f) $\begin{array}{r} 701 \\ \times 700 \\ \hline \end{array}$ (g) $\begin{array}{r} 527 \\ \times 856 \\ \hline \end{array}$ (h) $\begin{array}{r} 948 \\ \times 600 \\ \hline \end{array}$ (i) $\begin{array}{r} 856 \\ \times 208 \\ \hline \end{array}$ (j) $\begin{array}{r} 2629 \\ \times 9001 \\ \hline \end{array}$

4. Do the following division problems:

(a) $574\overline{)4016}$ (b) $25\overline{)72}$ (c) $19\overline{)162}$ (d) $151\overline{)1269}$

(e) $600\overline{)2007}$ (f) $790\overline{)8916}$ (g) $87\overline{)845}$ (h) $204\overline{)3015}$

5. Divide. Express answers to the nearer whole number.

(a) $507\overline{)31746}$ (b) $12\overline{)3677}$

31746 ÷ 507 = _____ 3677 ÷ 12 = _____

(to nearer whole number) (to nearer whole number)

6. Perform the following operations. (You should be able to do the calculations in your head.)

(a) $560 \div 70 =$ _____ (f) $6000 \times 30 =$ _____

(b) $1000 \div 10 =$ _____ (g) $20 \times 20 =$ _____

(c) $7 \times 9000 =$ _____ (h) $59000 \div 1000 =$ _____

(d) $27000 \div 3 =$ _____ (i) $2530 \div 10 =$ _____

(e) $270 \div 30 =$ _____ (j) $90 \times 10 =$ _____

Part B

1. Write in words. (Be sure to spell correctly and use commas and hyphens in the correct places.)

(a) 85,960

eighty-five thousand, nine hundred sixty

(b) 760,135

(c) 10,000,000

(d) 5,220,006

(e) 991,307

2. Write in numerals:

(a) Fifty thousand, ninety-six

50,096

(b) One hundred thousand, seventeen

(c) Six hundred ninety thousand, two

(d) Three hundred seventy-eight thousand, five hundred thirty

(e) Two hundred million, five

(f) Five hundred twenty million, thirty-two thousand, forty

3. From the numeral 76,843,201 give the place value of each digit.

7 ten million

6

8

4

3

2

0

1

4. By using the numeral 50271003048 answer the following questions:

   (a) Divide the numeral into periods by using commas.

   _____

   (b) The digits in the units period are _____.

   (c) 50 is in the _____ period.

   (d) 271 is in the _____ period.

   (e) What digits are in the thousands period? _____

   (f) Write out the numeral in words.

   _____

   _____

   _____

5. Round the following numerals to the place value indicated:

   (a) 98,901; to the nearer thousand 99,000

   (b) 925; to the nearer ten _____

   (c) 1,999; to the nearer hundred _____

   (d) 201,350; to the nearer thousand _____

   (e) 1,555,657; to the nearer hundred thousand _____

   (f) 45,982; to the nearer hundred _____

   (g) 1,854; to the nearer ten _____

   (h) 9,325; to the nearer thousand _____

Part C

1. Name each numeral in the following operations. (The names to choose from are: addend, sum, subtrahend, difference, minuend, factor, product, quotient, dividend, divisor, remainder.)

$$\begin{array}{r} 19 \\ -\ 6 \\ \hline 13 \end{array} \qquad \begin{array}{r} 8 \\ +4 \\ \hline 12 \end{array} \qquad \begin{array}{r} 15 \\ \times\ 3 \\ \hline 45 \end{array} \qquad \begin{array}{r} 37 \\ 9\overline{)334} \\ 27 \\ \hline 64 \\ 63 \\ \hline 1 \end{array}$$

19	minuend
6	
13	
8	
4	
12	
15	
3	
45	
9	
334	
37	
1	

2. Match the following by placing the appropriate letter in each blank:

(c)	$12 + 0 = 12$	(a) Multiplication by 0
	$3 \times 6 = 6 \times 3$	(b) Commutative Property of Addition
	$19 \times 0 = 0$	(c) Adding 0
	$(2+7) + 6 = 2 + (7+6)$	(d) Distributive Property
	$14 + 2 = 2 + 14$	(e) Commutative Property of Multiplication
	$21 \times 1 = 21$	(f) Associative Property of Addition
	$(9+2) \times 3 = 9 \times 3 + 2 \times 3$	(g) Multiplication by 1
	$(7 \times 8) \times 4 = 7 \times (8 \times 4)$	(h) Associative Property of Multiplication

3. Fill in the following blanks with the appropriate answers. (Remember that division by zero is impossible.)

(a) Any whole number multiplied by zero gives _____.

(b) Zero divided by any whole number (except zero) gives a quotient of _____.

(c) 17 - 17 = _____

(d) Division by zero is _____.

(e) a ÷ 0 = _____

(f) Any whole number divided by one gives _____ as the quotient.

(g) 0 ÷ 5 = _____ (j) 67 × 1 = _____ (m) 21 ÷ 1 = _____

(h) 15 - 0 = _____ (k) 51 ÷ 51 = _____ (n) 15 × 0 = _____

(i) 0 + 15 = _____ (l) 29 ÷ 0 = _____ (o) 0 × 509 = _____

Part D

For each of the following pairs of numbers:

(i) write each number as the product of primes (If the number is prime, write "prime" in the blank.)
(ii) state the highest common factor (H.C.F.) for each pair
(iii) list three multiples of each number
(iv) state the lowest common multiple for each pair

(a) 5 and 15

(i) prime factors

5: prime 15: 3 x 5

(ii) H.C.F. is 5.

(iii) Multiples of 5 are 5, 10, 15

Multiples of 15 are 15, 30, 45

(iv) L.C.M. = 3 (once) x 5 (once) = 3 x 5 = 15
(The L.C.M. can also be found by looking at the list of multiples.)

(b) 12 and 20

(i) 12: ____ × ____ × ____ 20: ____ × ____ × ____ } prime factors

(ii) H.C.F. = _____

(iii) Multiples of 12 are _____.

Multiples of 20 are _____.

(iv) L.C.M. = _____.

(c) 9 and 15

(i) prime factors

9: _____ 15: _____

(ii) H.C.F. is _____.

(iii) Multiples of 9 are _____.

Multiples of 15 are _____.

(iv) L.C.M. = _____.

(d) 24 and 36

(i) prime factors

24: _____ 36: _____

(ii) H.C.F. is _____.

(iii) Multiples of 24 are _____.

Multiples of 36 are _____.

(iv) L.C.M. = _____.

Part E

1. A table of distances between important cities is given below. The time taken to travel these distances by train and by aeroplane are given. Find the average kilometres per hour by train, and by aeroplane, for each trip. (Round each answer to the nearer whole number.) Show calculations below.

	Kilometres	Hours by Train	Average Kilometres per hour by Train	Hours by Plane	Average Kilometres per hour by Plane
Montreal - Toronto	558	6	93	1	558
Montreal - Vancouver	4802	71		10	
Toronto - Vancouver	4507	67		8	
Winnipeg - Calgary	1387	13		3	
Toronto - New York	897	12		1	

2. Solve each of the following problems.
Your answers should consist of two parts:
(i) an estimation of the answer using rounded numbers.
(ii) finding the actual answer from the given numbers. (Answers should be rounded to the nearer whole number.)

(a) Last year a journeyman plasterer received $13,540 in wages. If the total number of days worked was 225, what was the plasterer's average wage per day.

(i) Estimation:

(ii) Actual answer:

(b) On a certain job a journeyman roofer and an applicant apprentice took 35 hours to complete the work. Using a wage rate of $6.50 per hour for the journeyman and $4.70 per hour for the applicant apprentice, find the wage received by each man. How much more did the journeyman earn than the apprentice earned? (Use $6.50 = 650¢ and $4.70 = 470¢ To find the answer in dollars, divide by 100.)

(i) Estimation:

(ii) Actual Answer:

(c) A contractor is hired to build a house which has a floor area of 1276 square feet. How much would the contractor be paid if he builds for $20 per square foot? What would be the cost of the house if the lot cost $11,000?

(i) Estimation:

(ii) Actual Answer:

(d) Throughout the year a pipefitter had $2954 income tax deducted from his wages. In February the pipefitter was computing his income tax return and found he should have paid only $2309 income tax for the year. How much of an income tax rebate should the pipefitter expect to receive?

(i) Estimation:

(ii) Actual Answer:

End of Lesson 1

Practical Mathematics
Module 2
Lesson 2

WHAT IS A FRACTION?

Topic One: Meaning of Fractions

A. Consider the whole numbers 3 and 4. When we multiply these numbers together, their product (or result) is another whole number.
e.g. $(3 \times 4 = 12)$. But suppose we take the same two numbers and divide them; the result will not be a whole number.

Divide 3 by 4, the result is $\frac{3}{4}$. Divide 4 by 3, the result is $\frac{4}{3}$.

1. Give the result when:

   (a) 7 is divided by 9 $7 \div 9 =$ $\frac{7}{9}$

   (b) 15 is divided by 8 $15 \div 8 =$ _____

   (c) 5 is divided by 6 _____

   (d) 10 is divided by 27 _____

   (e) 36 is divided by 9 _____

   In each of the above, note that the result when two numbers are divided is not necessarily a whole number but rather a fraction.

> A FRACTION is the name we use to represent numbers such as $\frac{3}{4}$ and $\frac{4}{3}$.

B. Consider the fraction $\frac{3}{4}$ which is read "3 over 4" or "3 quarters".
This fraction can have several meanings:

1. A quotient - This is the result when two whole numbers are divided.
   e.g. $3 \div 4$. The result is the fraction $\frac{3}{4}$.

Give the fraction which is represented by each of the following quotients:

(a) $5 \div 2$ _____ (c) $6 \div 1$ _____

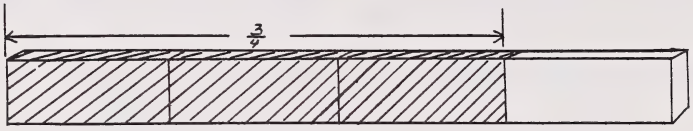
(b) $7 \div 9$ _____ (d) $4 \div 7$ _____

In each of the above, the fraction tells how many times as much, or what part, one number is of the other.

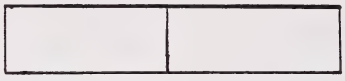
2. A fraction is the representation of one or more of the equal parts of one unit.

EXAMPLE: $\frac{3}{4}$ of a length of lumber means that the length of lumber has been cut into 4 equal parts and 3 of these parts have been taken. (Figure 2-1, shows how the length of lumber has been cut. The shading illustrates 3 of the 4 equal parts.)

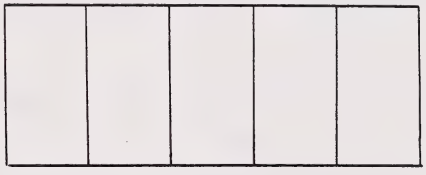
Fig. 2-1



PROBLEM 1: If a unit is divided into 2 equal parts, each of the parts is $\frac{1}{2}$. Shade $\frac{1}{2}$ of the diagram below.



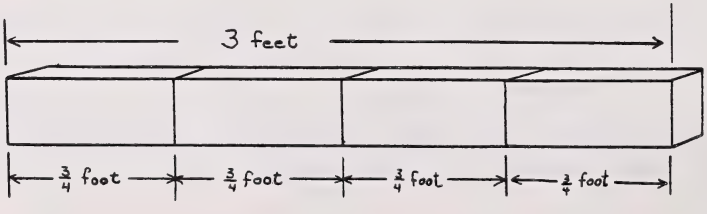
PROBLEM 2: If a unit is divided into 5 equal parts, each of the equal parts is $\frac{1}{5}$. Shade $\frac{2}{5}$ of the diagram below.



3. A fraction is the representation of one of the equal parts into which a given quantity has been divided.

EXAMPLE 1: $\frac{3}{4}$ feet is the length of each piece of lumber which has been cut from a board which was 3 feet long and was cut into 4 equal pieces. (See Figure 2-2.)

Fig. 2-2



EXAMPLE 2: $\frac{3}{4}$ means that 3 is divided into 4 equal parts of $\frac{3}{4}$ units each. If \$3 is to be divided equally among 4 persons, the amount that each receives in dollars is $3 \div 4$ or $\frac{3}{4}$. (You have \$3 and you are dividing it into 4 equal parts.)

PROBLEM 1: You receive your monthly check of \$550. Each month you plan on putting equal amounts into two separate accounts in the bank. How much, in dollars, would you put into each account? (Give the answer as a fraction.)

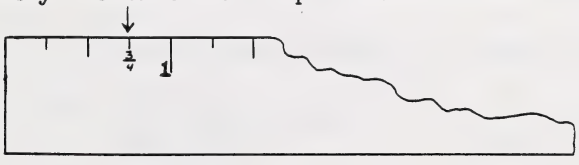
Solution: _____

PROBLEM 2: A piece of lumber 5 feet long is to be divided equally into 8 parts. What is the length, in feet, of each part?

Solution: _____

4. A fraction is merely the label for a point on a ruler.

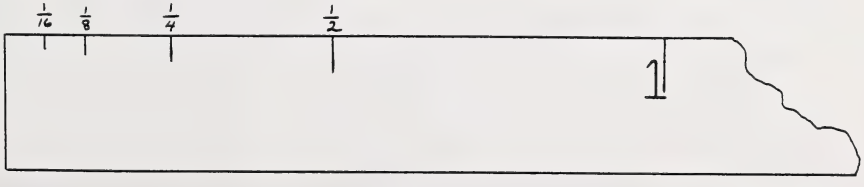
EXAMPLE:



The point marked on the ruler is $\frac{3}{4}$ inch.

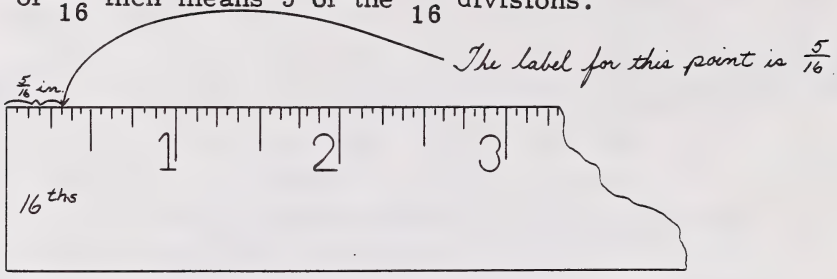
A ruler scale is an example of a linear scale. A linear scale is divided evenly in order to measure length. The major divisions may be divided in half, and half again, and half again, to give fractions of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, and so on.

In the enlarged section of a scale shown below, a measure of 1 inch is divided in half to give $\frac{1}{2}$, $\frac{1}{2}$ is divided in half to give $\frac{1}{4}$, $\frac{1}{4}$ in half to give $\frac{1}{8}$, $\frac{1}{8}$ in half to give $\frac{1}{16}$.



A common division for an ordinary ruler is 16ths $\left(\frac{1}{16}\right)$ of an inch.

A measure of $\frac{5}{16}$ inch means 5 of the $\frac{1}{16}$ divisions.



EXERCISE: On the following ruler diagram, locate and mark the following points ('a' and 'd' are done for you).

(a) $\frac{1}{2}$ inch

(b) $\frac{7}{8}$ inch

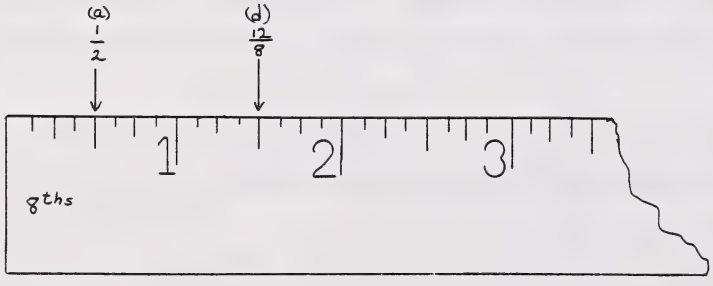
(c) $\frac{7}{4}$ inch

(d) $\frac{12}{8}$ inch

(e) $\frac{13}{4}$ inch

(f) $\frac{6}{4}$ inch

(g) $\frac{0}{3}$ inch



Topic Two: Numerator and Denominator of a Fraction

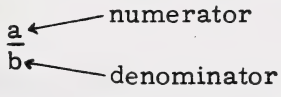
All fractions have a standard form. They are written as one number above another with a horizontal bar between. Some examples of fractional numbers are:

$\frac{3}{4}, \frac{4}{3}, \frac{36}{9}, \frac{5}{8}, \frac{7}{4}, \frac{0}{3}$

> In general, a COMMON FRACTION is a number of the form $\frac{a}{b}$ (read "a over b") where "a" is a whole number and "b" is any whole number except zero. "b" cannot equal zero since division by zero is impossible.

Give the names of any five fractional numbers. _____

In any fraction $\frac{a}{b}$, 'a' is the NUMERATOR (the number above the line) and 'b' is the DENOMINATOR (the number below the line).



The numerator of the fraction $\frac{1}{2}$ is _____.

Complete the following table:

Fraction	Numerator	Denominator
$\frac{1}{5}$	1	5
$\frac{9}{8}$	9	____
$\frac{7}{8}$	____	____
$\frac{5}{25}$	____	25
$\frac{2}{5}$	____	____

When a fraction is thought of as a quotient, the number above the line (the _____) is the value being divided into, the number below the line (the _____) is the value doing the dividing. For example, we may think of the fraction $\frac{6}{2}$ as being the quotient $6 \div 2$ (or 3).

Topic Three: Types of Fractions

A. Proper Fractions

A proper fraction is a fraction which expresses a quantity less than 1. As a measure, if a fraction indicates a measure less than 1, the fraction is a proper fraction.

> Fractions in which the numerators are less than the denominators (such as $\frac{3}{4}$ and $\frac{2}{5}$) refer to amounts which are less than one unit and are called PROPER FRACTIONS.

Give three examples of proper fractions:

(a) _____ (b) _____ (c) _____

B. Improper Fractions

An improper fraction expresses a quantity equal to one or greater than one. As a measure, would an improper fraction indicate a measure less than one, equal to one, and/or greater than one?

_____

> Fractions in which the numerators are equal to or are greater than the denominators (such as $\frac{5}{5}$ and $\frac{4}{3}$) refer to amounts that are equal to or are greater than one unit and are called IMPROPER FRACTIONS.

1. Give three examples of improper fractions:

   (a) _____ (b) _____ (c) _____

2. Fill in the blanks:

   (a) In a proper fraction, the _____ is greater than the _____.

   (b) In an improper fraction, the _____ is greater than, or equal to the _____.

(c) The value of an improper fraction is _____ than or equal to 1. The value of a proper fraction is _____ than 1.

Consider the fractions $\frac{5}{5}$, $\frac{10}{5}$, $\frac{8}{4}$, and $\frac{12}{3}$. Each of these improper fractions can also be written as a whole number.

EXAMPLE 1: $\frac{5}{5} = 5 \div 5 = 1$

EXAMPLE 2: $\frac{10}{5} = 10 \div 5 = 2$

The top number of each fraction can be divided evenly by the bottom number.

> Improper fractions in which the denominator divides evenly into the numerator are whole numbers.

3. Rewrite the following as whole numbers:

(a) $\frac{8}{4} =$

(b) $\frac{12}{3} =$

(c) $\frac{0}{6} = 0$

(d) $\frac{4}{4} =$

(e) $\frac{9}{9} =$

(f) $\frac{0}{2} =$

4. Give four examples of improper fractions which can be written as whole numbers and give the whole numbers.

(a) $\frac{12}{2} = 6$

(b) _____

(c) _____

(d) _____

5. Complete the chart below.

Fraction	Kind of Fraction	Whole Number (if any)
(a) $\frac{3}{8}$	Proper	_____
(b) $\frac{15}{16}$	Proper	_____
(c) $\frac{16}{4}$	Improper	_____
(d) $\frac{9}{22}$	_____	_____
(e) $\frac{31}{31}$	_____	_____
(f) $\frac{24}{6}$	_____	_____
(g) $\frac{1}{2}$	_____	_____
(h) $\frac{6}{7}$	_____	_____
(i) $\frac{18}{6}$	_____	_____

Topic Four: Mixed Numbers

Some numbers have both a whole-number part and a fractional part. Numbers of this type are called MIXED NUMBERS.

EXAMPLE: $2\frac{2}{3}$ is a mixed number which is composed of the whole number 2 and the common fraction $\frac{2}{3}$. It represents the sum of 2 and $\frac{2}{3}$. ($2\frac{2}{3}$ is read "two and two thirds".)

$$2\frac{2}{3} = 2 + \frac{2}{3}$$

Express the following mixed numbers as the sum of two numbers.

(a) $5\frac{2}{3} =$ _____ (b) $1\frac{3}{8} =$ _____ (c) $15\frac{9}{11} =$ _____

An improper fraction is sometimes changed to a mixed or whole number, or also, a mixed or whole number may be changed to an improper fraction.

A. Changing a Whole Number to an Improper Fraction

EXAMPLE 1: Change 1 to a fraction with denominator 6.

$$1 = \frac{6}{6} \quad \text{Check: } 6 \div 6 = 1$$

EXAMPLE 2: Change 5 to a fraction with denominator 6.

$$5 = \frac{5 \times 6}{6} = \frac{30}{6} \quad \text{Check: } 30 \div 6 = 5$$

Multiply the whole number (5) by the given denominator (6). Place the result (30) over the given denominator.

To change a whole number to an improper fraction of a given denominator, multiply the whole number by the given denominator and place the result over the common denominator.

Using the given denominators, find fractions for the following. (Show the inbetween step). Check your results by dividing.

	Check
(a) $4 = \frac{4 \times 6}{1 \times 6} = \frac{24}{6}$	$24 \div 6 = 4$
(b) $5 = \frac{5 \times 5}{1 \times 5} = \frac{\quad}{5}$	___ $\div 5 = 5$
(c) $7 = \frac{\quad}{\quad} = \frac{\quad}{5}$	___ $\div 5 = 7$
(d) $10 = \frac{\quad}{\quad} = \frac{\quad}{9}$	_____

(e) $9 = ____ = \frac{\quad}{9}$ _____

(f) $1 = ____ = \frac{\quad}{100}$ _____

B. Changing a Mixed Number to an Improper Fraction

EXAMPLE: $4\frac{3}{5} = 4 + \frac{3}{5} = \frac{20}{5} + \frac{3}{5} = \frac{23}{5}$

(i) Express the numbers as the sum of two numbers.
(ii) Rewrite the whole number as a fraction with the given denominator.
(iii) Add the numerator of the proper fraction to the numerator of the improper fraction. Express the sum over the given denominator.

1. Complete the following chart showing all steps.
This is a self-correcting exercise. Do your best in completing the chart then check your results with the results on page 29 of this lesson.

Mixed Number	Intermediate Steps	Improper Fraction
(a) $4\frac{3}{4}$	$4 + \frac{3}{4} = \frac{16}{4} + \frac{3}{4}$	$\frac{\quad}{4}$
(b) $7\frac{1}{4}$	$7 + \frac{1}{4} = \frac{\quad}{4} + \frac{\quad}{4}$	$\frac{\quad}{4}$
(c) $8\frac{2}{3}$	_____	_____
(d) $4\frac{1}{5}$	_____	_____
(e) $2\frac{4}{9}$	_____	$\frac{22}{9}$
(f) $9\frac{1}{2}$	_____	_____
(g) $5\frac{5}{7}$	_____	_____
(h) $5\frac{3}{8}$	_____	_____
(i) $2\frac{5}{16}$	_____	_____
(j) $3\frac{7}{16}$	_____	_____

Once you understand the steps in changing a mixed number to an improper fraction, a shorter method can be applied. Study the following examples:

EXAMPLE 1: $4\frac{1}{9} = \frac{(4 \times 9) + 1}{9} = \frac{37}{9}$

EXAMPLE 2: $7\frac{3}{11} = \frac{(7 \times 11) + 3}{11} = \frac{80}{11}$

2. Find the improper fraction equivalent to each of the following by using the "short-cut".

a. $1\frac{1}{2} = \frac{(1 \times 2) + 1}{2} = $ _____

b. $5\frac{1}{8} = \frac{(5 \times 8) + 1}{8} = $ _____

c. $7\frac{3}{5} = $ _____ = _____

d. $4\frac{4}{5} = $ _____ = _____

e. $10\frac{11}{12} = $ _____ = _____

C. Changing an Improper Fraction to a Mixed Number

Since this is a reverse procedure from what we have just studied, we should be able to reverse the steps and find the improper fraction equivalent to a mixed number.

EXAMPLE: $\frac{23}{5} = \frac{20}{5} + \frac{3}{5} = 4 + \frac{3}{5} = 4\frac{3}{5}$

(i) Express the mixed number as the sum of an improper fraction (which can be written as a whole number) and a proper fraction. Both fractions must have the same denominator.

(ii) Express this sum as the sum of a whole number and a proper fraction.

(iii) Combine the whole number and the proper fraction into a mixed number.

Change the following fractions to mixed numbers. Please show all your work.

The answers to this exercise can be found on page 30 of this lesson.

Improper Fraction	Intermediate Steps	Mixed Number
a. $\frac{20}{3}$	3 divides evenly into 18: $\frac{18}{3} + \frac{2}{3} = 6 + \frac{2}{3}$	$6\frac{2}{3}$
b. $\frac{17}{9}$	9 divides evenly into 9: $\frac{9}{9} + \frac{8}{9} = __ + __$	_____
c. $\frac{27}{8}$	_____	_____
d. $\frac{5}{2}$	_____	_____
e. $\frac{31}{16}$	_____	_____
f. $\frac{19}{6}$	_____	_____

In changing an improper fraction to a whole number or a mixed number we can use the fact that a fraction is a quotient.

EXAMPLE 1: $\frac{32}{4} = 32 \div 4 = 8$

EXAMPLE 2: $\frac{47}{9} = 47 \div 9 = 5$, remainder $2 = 5\frac{2}{9}$

From the above examples, we can draw the following conclusions:

1. An improper fraction can be written as a whole number whenever its denominator divides evenly into the numerator.

   EXAMPLES:

   $\frac{48}{8} = 48 \div 8 = 6$

   $\frac{64}{4} = 64 \div 4 = 16$

   } Results are whole numbers.

2. An improper fraction can be written as a mixed number whenever a remainder occurs in the division process. The whole-number part of the mixed number is the quotient obtained when you divide. The fractional part is obtained by placing the remainder over the divisor.

EXAMPLES:

$\frac{35}{4} = 35 \div 4 = 8$, remainder $3 = 8\frac{3}{4}$ (3 ← remainder, 4 ← divisor, 8 ← quotient)

$\frac{82}{7} = 82 \div 7 = 11$, remainder $5 = 11\frac{5}{7}$

Results are mixed numbers.

Change each improper fraction to a whole number or mixed number by using division.

(a) $\frac{85}{9} = 85 \div 9 = 9$, remainder $4 = 9\frac{4}{9}$

(b) $\frac{62}{7} = 62 \div 7 =$

(c) $\frac{36}{3} =$

(d) $\frac{12}{11} =$

(e) $\frac{50}{5} =$

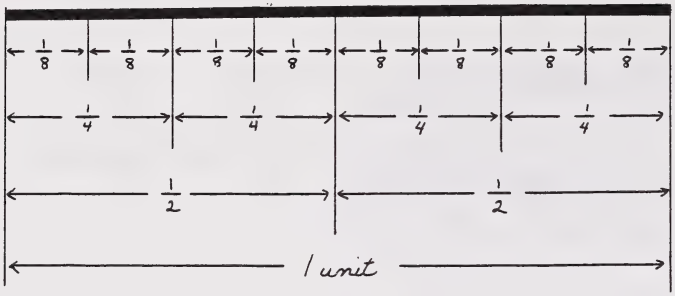
(f) $\frac{9}{8} =$

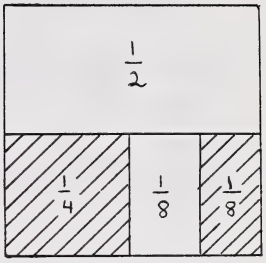
(g) $\frac{121}{10} =$

(h) $\frac{75}{19} =$

Topic Five: Equivalent Fractions

A. Finding Equivalent Fractions

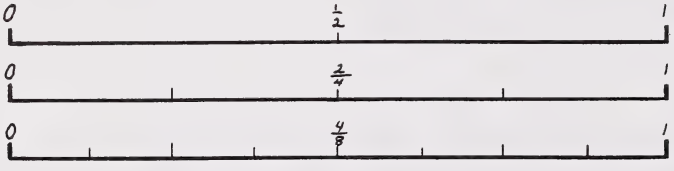




If a unit is divided into 2 equal parts, each of the parts is $\frac{1}{2}$.

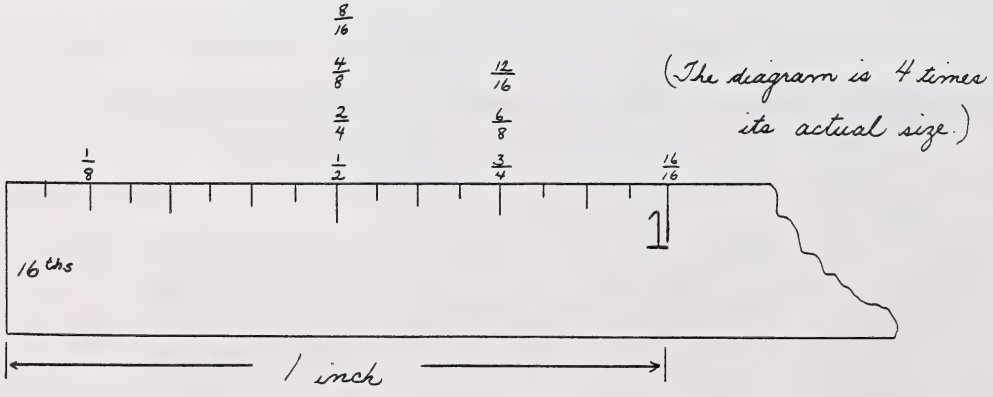
If this same unit is divided into 4 equal parts, each part is $\frac{1}{4}$.
From the diagrams we can see that $\frac{1}{2} = \frac{2}{4}$ and $\frac{2}{4} = \frac{4}{8}$. Similarly $\frac{3}{4} = \frac{6}{8}$.

Let's consider another diagram to see how these fractions compare.



In the diagram you should see that $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ have the same value.
Fractions which have the same value are called EQUIVALENT FRACTIONS.

Refer to the ruler diagram below in answering the questions on page 15.



(a) Two other names for $\frac{3}{4}$ inch are $\frac{12}{16}$ inch and _____ inch.

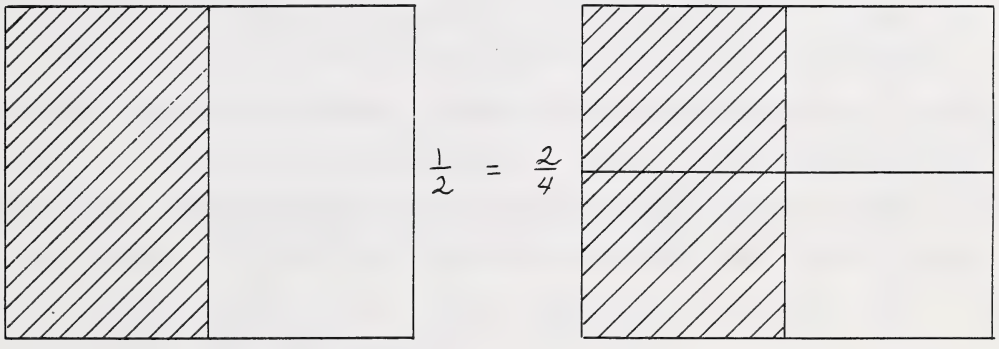
(b) Two other names for $\frac{1}{2}$ inch are _____ inch and _____ inch.

(c) Another name for $\frac{1}{8}$ inch is _____ inch.

(d) Fractions which name the same measure are said to be _____ fractions.

Two equivalent fractions can be thought of as representing the same part of a region.

EXAMPLE: A piece of paper is cut into halves and then quarters as shown in the diagrams below. (The left half of the original piece is shaded.)



Looking at the diagrams, you can see that by dividing the piece of paper a second time you have doubled the total number of pieces (2 pieces become 4) and doubled the number of shaded pieces (1 piece becomes 2).

In the first diagram, the shaded region can be represented by the fraction $\frac{1}{2}$ (since 1 out of the 2 parts is shaded). In the diagram, the shaded region can be represented by the fraction $\frac{2}{4}$ since 2 out of the 4 parts are shaded. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions since they represent the same shaded region.

$$\text{i.e. } \frac{1}{2} = \frac{2}{4}$$

$\frac{1}{2}$ can be written in the equivalent form $\frac{2}{4}$ if both its numerator and denominator are multiplied by 2.

$$\text{i.e. } \frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

In general, the value of a fraction is unchanged if both its terms are multiplied by the same number.

> To find a fraction equivalent to a given fraction, we may multiply the numerator and denominator of the given fraction by the same number (except zero).

For each of the following fractions, give two equivalent fractions. Show how you obtain your fractions.

(a) $\frac{1}{2}$ $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ $\frac{1 \times 8}{2 \times 8} = \frac{8}{16}$

(b) $\frac{5}{4}$ $\frac{5 \times 3}{4 \times 3} =$ $\frac{5 \times 7}{4 \times 7} =$

(c) $\frac{3}{8}$

(d) $\frac{3}{2}$

(e) $\frac{3}{4}$

(f) $\frac{5}{6}$

(g) $\frac{6}{1}$

(h) $\frac{1}{4}$

(i) $\frac{2}{3}$

(j) $\frac{8}{5}$

Let's consider a reverse procedure; combining equal parts to form larger parts.

EXAMPLE: A person has 6 squares of paper of the same size. Three of these squares are shaded. He tapes the 3 shaded pieces together and the 3 unshaded pieces together.

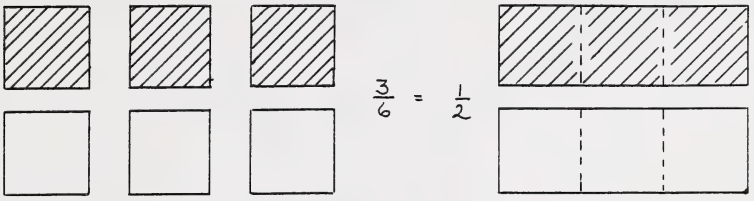


Diagram 1 Diagram 2

Looking at the two diagrams, you can see that when the squares are joined, the total number of pieces is divided by 3 (6 pieces become 2) and the total number of shaded pieces is also divided by 3 (3 pieces become 1).

In the first diagram, the shaded region can be represented by the fraction $\frac{3}{6}$ (since 3 out of the 6 parts are shaded). In the second diagram, the shaded region can be represented by the fraction $\frac{1}{2}$ (since 1 out of the two parts are shaded). $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions since they represent the same shaded region.

$$\text{i.e. } \frac{3}{6} = \frac{1}{2}$$

$\frac{3}{6}$ can be written in the equivalent form $\frac{1}{2}$ if both its numerator and denominator are divided by 3.

$$\text{i.e. } \frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

In general, the value of a fraction is unchanged if both its terms are divided by the same number.

> To find a fraction equivalent to a given fraction, we may divide the numerator and denominator of the given fraction by the same number (except zero).

For each of the following fractions, find an equivalent fraction by dividing both terms by the same number.

(a) $\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

(b) $\frac{8}{12} =$

(c) $\frac{10}{4} = \frac{10 \div 2}{4 \div 2} =$

(d) $\frac{9}{6} =$

(e) $\frac{4}{6} =$

(f) $\frac{10}{15} =$

(g) $\frac{3}{9} =$

(h) $\frac{6}{10} =$

B. Raising Fractions to Higher Terms

The numerator and denominator of a fraction are called "terms." When both the numerator and the denominator (the terms) of a fraction are multiplied by the same number (except 1) the fraction which results is said to be in higher terms.

1. Change each of the following to equivalent fractions with denominator 16 as in the example. Show all steps.

EXAMPLE: $\frac{3}{4} = \frac{3 \times 4}{4 \times 4}$

$= \frac{12}{16}$

To obtain the new denominator 16, the original denominator, 4, must be multiplied by 4. Therefore, the original numerator, 3, must be multiplied by 4 to obtain the new numerator.

(a) $\frac{1}{2} = \frac{1 \times 8}{2 \times 8} = \frac{\quad}{16}$

(b) $\frac{5}{8} = \frac{5 \times}{8 \times} = \frac{\quad}{16}$

(c) $\frac{3}{8} = \frac{3 \times}{8 \times} = \frac{\quad}{16}$

(d) $\frac{7}{8} = \quad =$

(e) $\frac{1}{4} = \quad =$

2. Express each of the following fractions as a mixed number in which the fractional part of the number is in 32nds.
(The first one is done for you.)

(a) $\frac{15}{2} = \frac{14}{2} + \frac{1}{2} = 7\frac{1}{2} = 7\ \frac{1 \times 16}{2 \times 16} = 7\frac{16}{32}$

Change $\frac{1}{2}$ to 32nds by multiplying both terms by 16

(b) $\frac{15}{4} =$

(c) $\frac{17}{4} =$

(d) $\frac{19}{8} =$

(e) $\frac{72}{16} =$

(f) $\frac{38}{8} =$

3. Give the next three fractions for each of the following.

(a) $\frac{1}{5}, \frac{2}{10}, \frac{3}{15},$ _____, _____, _____

(You are multiplying the first fraction by $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}$, etc.)

(b) $\frac{5}{6}, \frac{10}{12}, \frac{15}{18},$ _____, _____, _____

(c) $\frac{5}{8}, \frac{10}{16}, \frac{15}{24},$ _____, _____, _____

(d) $\frac{0}{2}, \frac{0}{4}, \frac{0}{6},$ _____, _____, _____

(e) $\frac{2}{3}, \frac{4}{6}, \frac{6}{9},$ _____, _____, _____

C. Reducing Fractions to Lower Terms

When both the numerator and the denominator, (the terms) of a fraction are divided by the same factor (except 1) the resulting fraction is said to be in lower terms.

I. Factor

You may be asking, "What is a factor?" Remember! The word "factors" refers to the numbers used when we multiply. When we say that $3 \times 2 = 6$, the 3 and 2 are factors of 6, and 6 is the product.

$$\begin{array}{cccccc} \text{Factor} & \times & \text{factor} & = & \text{product} \\ 3 & \times & 2 & = & 6 \end{array}$$

$$\begin{array}{cccccc} \text{or,} & \text{product} & \div & \text{factor} & = & \text{factor} \\ & 6 & \div & 3 & & 2 \end{array}$$

$$\begin{array}{cccccc} \text{also,} & 6 & \div & 2 & = & 3. \end{array}$$

You could notice that 3 divides 6 and also that 2 divides 6. We say that 2 and 3 are factors of 6. Since 2 and 3 divide 6 exactly, then they are factors of 6.

> If a number A divides a number B, we say that A is a factor of B.

EXERCISE:

Give all the factors of the following numbers.

(a) 9; factors are 1, 3, and 9
(Ask yourself which numbers will divide evenly into 9.)

(b) 27; factors are 1, , , 27

(c) 16; factors are 1, 2, , , 16

(d) 24; factors are _____

(e) 35; factors are _____

(f) 50; factors are _____

NOTE: 1 is a factor of all numbers.

II. Prime Factors

A PRIME NUMBER is any non-zero whole number with no factors other than itself and 1.

e.g. The only factors of 19 are 1 and 19. Therefore, 19 is a prime number.

Whole numbers (other than 0 and 1) which are not primes are known as COMPOSITE NUMBERS. That is, a COMPOSITE NUMBER is any non-zero whole number with factors other than itself and 1. Every composite number can be written as the product of prime numbers. (Review page 13, Module 1, Lesson 5.)

e.g. 36 has the factors 1, 2, 3, 4, 6, 9, 12, 18, and 36. Hence, 36 is a composite number. 36 can be written as the product of prime numbers as follows:

$36 = 2 \times 18 = 2 \times (2 \times 9) = 2 \times 2 \times (3 \times 3)$

composite number; 2 and 3 are prime numbers

EXERCISE:

For each of the composite numbers below, express it as the product of prime numbers. If the number is prime, write "prime" beside it.

(a) 15 _____	(b) 11 _____	(c) 20 _____
(d) 24 _____	(e) 9 _____	(f) 7 _____
(g) 29 _____	(h) 32 _____	(i) 49 _____
(j) 16 _____	(k) 50 _____	(l) 23 _____

III. Common Factors

We are often interested in comparing the prime factors of two different numbers.

EXAMPLE 1:

$30 = 3 \times 2 \times 5$ (The prime factors are 2, 3 and 5.)
$21 = 3 \times 7$ (The prime factors are 3 and 7.)

3 is a factor of 30 and 3 is a factor of 21. Thus, 3 is a COMMON FACTOR of 21 and 30.

If we were reducing the fraction $\frac{21}{30}$ to lower terms, we would divide the numerator and the denominator by the factor 3.

$$\frac{21 \div 3}{30 \div 3} = \frac{7}{10}$$

EXAMPLE 2:

$70 = (2 \times 5) \times 7$ (The prime factors are 2, 5, and 7.)
$110 = (2 \times 5) \times 11$ (The prime factors are 2, 5, and 11.)

2 and 5 are common factors of 70 and 110.
Thus, (2×5) or 10 is also a common factor.

We could reduce the fraction $\frac{70}{110}$ to lower terms by dividing both its terms by 2, 5, or 10.

i.e. $\frac{70}{110} = \frac{70 \div 2}{110 \div 2} = \frac{\quad}{\quad}$; $\frac{70}{110} = \frac{70 \div 5}{110 \div 5} = \frac{\quad}{\quad}$; $\frac{70}{110} = \frac{70 \div 10}{110 \div 10} = \frac{\quad}{\quad}$

Thus, three equivalent fractions that are in lower terms than $\frac{70}{110}$ are $\frac{\quad}{\quad}$, $\frac{\quad}{\quad}$, and $\frac{\quad}{\quad}$.

EXERCISE:

1. Find the prime factors of the numerator and denominator of each fraction and list all the common prime factors. Then give an equivalent fraction in lower terms by dividing both terms by a common factor.

Fraction	Prime Factors of Numerator and Denominator	Common Prime Factors	Equivalent Fraction (Lower Terms)
$\frac{15}{35}$	$15 = 3 \times 5$ $35 = 5 \times 7$	5	$\frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}$
$\frac{20}{24}$	$20 = 2 \times 2 \times 5$ $24 = 2 \times 2 \times 2 \times 3$	2, 2	$\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$ (You could have also divided by 2.)
$\frac{39}{65}$	39 = ____ 65 = ____	____	____
$\frac{20}{30}$	20 = ____ 30 = ____	____	____
$\frac{56}{96}$	56 = ____ 96 = ____	____	____
$\frac{24}{36}$	24 = ____ 36 = ____	____	____
$\frac{75}{50}$	75 = ____ 50 = ____	____	____
$\frac{63}{56}$	63 = ____ 56 = ____	____	____

2. Use this set of equivalent fractions in answering the following questions.

$$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}\right\}$$

(a) Give all the fractions that are in lower terms than $\frac{15}{20}$.

_____

(b) What fraction is in lower terms than any other? _____

3. Complete this list of equivalent fractions.

$$\left\{\text{_____}, \text{_____}, \frac{6}{9}, \frac{8}{12}, \text{_____}, \text{_____}\right\}$$

(a) Give three fractions that are in lower terms than $\frac{8}{12}$.

_____

(b) Give three fractions that are in higher terms than $\frac{6}{9}$.

_____

(c) What fraction from (a) and (b) is in lower terms than any other fraction?

_____

4. Change each of the following fractions to an equivalent fraction with the denominator indicated:

(a) $\frac{4}{3} = \frac{\quad}{3 \times 4} = \frac{\quad}{12}$

(b) $\frac{5}{6} = \frac{5 \times 6}{\quad} = \frac{\quad}{36}$

(c) $\frac{7}{4} = \frac{\times 4}{\times 4} = \frac{\quad}{16}$

(d) $\frac{10}{3} = \frac{\quad}{\quad} = \frac{\quad}{30}$

(e) $\frac{90}{60} = \frac{\quad}{\quad} = \frac{\quad}{2}$

(f) $\frac{3}{5} = \frac{\quad}{\quad} = \frac{\quad}{20}$

(g) $\frac{7}{8} = \frac{\quad}{\quad} = \frac{\quad}{144}$

(h) $\frac{7}{28} = \frac{\quad}{28 \div 7} = \frac{\quad}{4}$

(i) $\frac{24}{64} = \frac{\quad}{\quad} = \frac{\quad}{8}$

(j) $\frac{49}{56} = \frac{\quad}{\quad} = \frac{\quad}{8}$

D. Reducing Fractions to Lowest Terms (or Simplest Terms)

I. Lowest Terms

Consider the equivalent fractions: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, $\frac{5}{15}$. Which one of these fractions is in lower terms than any other? _____

If you answered $\frac{1}{3}$ you are correct!
Let's examine $\frac{1}{3}$. The only factor that 1 and 3 have in common is 1.
Therefore, $\frac{1}{3}$ cannot be reduced to lower terms.

> When the numerator and the denominator of a fraction have no common factor other than 1, the fraction is in LOWEST TERMS.

EXAMPLE 1: Is $\frac{12}{15}$ in lowest terms? _____

12; prime factors are 2, 2, and 3.
15; prime factors are 3 and 5.
The fraction is not in lowest terms since 12 and 15 have a common factor of 3.

EXAMPLE 2: Is $\frac{8}{9}$ in lowest terms? _____

8; prime factors are 2, 2 and 2
9; prime factors are 3 and 3.
8 and 9 have no common factor therefore the fraction is in lowest terms.

EXERCISES:

1. Tell whether or not the following fractions are in lowest terms by writing "lowest" in the blank following those fractions which are in lowest terms. (The prime factors of the numerators and denominators are given.)

(a) $\frac{14}{15}$ (2, 7) (3, 5) _____

(b) $\frac{8}{12}$ (2, 2, 2) (2, 2, 3) _____

(c) $\frac{15}{28}$ (3, 5) (2, 2, 7) _____

2. Tell whether or not the following fractions are in lowest terms. Write "lowest" in the blank following those fractions which are in lowest terms, otherwise, leave a blank.

(a) $\frac{6}{10}$ _____ (e) $\frac{9}{24}$ _____

(b) $\frac{9}{15}$ _____ (f) $\frac{15}{20}$ _____

(c) $\frac{8}{15}$ _____ (g) $\frac{10}{21}$ _____

(d) $\frac{10}{18}$ _____ (h) $\frac{25}{30}$ _____

II. Reducing to Lowest Terms

> Finding the lowest-term fraction for a given fraction is called REDUCING TO LOWEST TERMS.

To write a fraction in lowest terms, we can keep dividing its numerator and denominator by common factors until no further division can take place.

EXAMPLE: $\frac{60}{90} = \frac{60 \div 10}{90 \div 10} = \frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

10 is a common factor of 60 and 90 (i.e. $\frac{60}{90} = \frac{6}{9}$)

3 is a common factor of 6 and 9 (i.e. $\frac{6}{9} = \frac{2}{3}$)

2 and 3 have no common factors except 1.

$\frac{60}{90}$ in lowest terms is $\frac{2}{3}$.

To make sure a fraction is in lowest terms, it is usually easiest to divide the numerator and the denominator of the fraction by the highest common factor (H.C.F.).

The highest common factor of numbers is found by taking the product of those prime factors which are common to the numbers.

Consider the example on page 25. In order to find the H.C.F. of 60 and 90, we can write their prime factors.

$$60 = 2 \times \underline{2} \times \underline{3} \times \underline{5}$$

$$90 = \underline{2} \times \underline{3} \times 3 \times \underline{5}$$

The H.C.F. of 60 and 90 is $2 \times 3 \times 5$ or 30.

To reduce $\frac{60}{90}$ to lowest terms we would divide 60 and 90 by 30.

$$\frac{60}{90} = \frac{60 \div 30}{90 \div 30} = \frac{2}{3}$$

Now let use use this same method to reduce some more fractions to lowest terms.

EXAMPLE 1: Reduce $\frac{18}{24}$ to lowest terms.

Solution

Since $18 = \underline{2 \times 3} \times 3$ and $24 = 2 \times 2 \times \underline{2 \times 3}$, the H.C.F. of 18 and 24 is 2×3 or 6.

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

EXAMPLE 2: Reduce $\frac{60}{84}$ to lowest terms.

Solution

$$\frac{60}{84} = \frac{60 \div 12}{84 \div 12} = \frac{5}{7}$$

Why did we divide 60 and 84 by 12?

EXERCISES:

1. Self-correcting

Reduce the following fractions to lowest terms. Please show your work. The answers to these exercises can be found on page 30 of this lesson. (The method of dividing by the H.C.F. is preferred.)

(a) $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

(b) $\frac{20}{60} =$

(c) $\frac{50}{75} =$

(d) $\frac{16}{20} =$

(e) $\frac{12}{30} =$

(f) $\frac{9}{36} =$

(g) $\frac{30}{48} =$

(h) $\frac{15}{70} =$

(i) $\frac{9}{21} =$

(j) $\frac{25}{75} =$

2. Reduce the following fractions to lowest terms. (Some are already in lowest terms. Beside them write "lowest terms".)

(a) $\frac{16}{28}$ =

(b) $\frac{5}{20}$ =

(c) $\frac{4}{10}$ =

(d) $\frac{8}{9}$ =

(e) $\frac{20}{12}$ =

(f) $\frac{16}{64}$ =

(g) $\frac{63}{81}$ =

(h) $\frac{8}{27}$ =

(i) $\frac{12}{30}$ =

(j) $\frac{8}{36}$ =

3. There are times when the lowest terms are not the most convenient. The machinist commonly expresses measurements in thousandths, even though they could be expressed in lower terms. Express each of the following in thousandths (i.e. with a denominator of 1000).

(a) $\frac{3}{100}$ =

(b) $\frac{7}{10}$ =

(c) $\frac{47}{100}$ =

(d) $\frac{100}{10,000}$ =

Answers to Self-Correcting Exercises on Page 10 of this Lesson

1.

Mixed Number	Intermediate Steps	Improper Fraction
a. $4\frac{3}{4}$	$4 + \frac{3}{4} = \frac{16}{4} + \frac{3}{4}$	$\frac{19}{4}$ since $\left(\frac{16 + 3}{4} = \frac{19}{4}\right)$
b. $7\frac{1}{4}$	$7 + \frac{1}{4} = \frac{28}{4} + \frac{1}{4}$ $\left(\frac{7 \times 4}{4}\right)$	$\frac{29}{4}$
c. $8\frac{2}{3}$	$8 + \frac{2}{3} = \frac{24}{3} + \frac{2}{3}$ $\left(\frac{8 \times 3}{3}\right)$	$\frac{26}{3}$
d. $4\frac{1}{5}$	$4 + \frac{1}{5} = \frac{20}{5} + \frac{1}{5}$	$\frac{21}{5}$ $\left(\text{since } \frac{20 + 1}{5} = \frac{21}{5}\right)$
e. $2\frac{4}{9}$	$2 + \frac{4}{9} = \frac{18}{9} + \frac{4}{9}$	$\frac{22}{9}$
f. $9\frac{1}{2}$	$9 + \frac{1}{2} = \frac{18}{2} + \frac{1}{2}$	$\frac{19}{2}$
g. $5\frac{5}{7}$	$5 + \frac{5}{7} = \frac{35}{7} + \frac{5}{7}$	$\frac{40}{7}$
h. $5\frac{3}{8}$	$5 + \frac{3}{8} = \frac{40}{8} + \frac{3}{8}$ $\left(\frac{5 \times 8}{8}\right)$	$\frac{43}{8}$
i. $2\frac{5}{16}$	$2 + \frac{5}{16} = \frac{32}{16} + \frac{5}{16}$	$\frac{37}{16}$
j. $3\frac{7}{16}$	$3 + \frac{7}{16} = \frac{48}{16} + \frac{7}{16}$	$\frac{55}{16}$

Answers to Self-Correcting Exercises on Page 12 of this Lesson

1.

Improper Fractions	Intermediate Steps	Mixed Numbers
a. $\frac{20}{3}$	$\frac{18}{3} + \frac{2}{3} = 6 + \frac{2}{3}$ (18 can be divided evenly by 3.)	$6\frac{2}{3}$
b. $\frac{17}{9}$	$\frac{9}{9} + \frac{8}{9} = 1 + \frac{8}{9}$ $\left(\frac{17 - 9}{9} = \frac{8}{9}\right)$	$1\frac{8}{9}$
c. $\frac{27}{8}$	$\frac{24}{8} + \frac{3}{8} = 3 + \frac{3}{8}$ (24 can be divided evenly by 8.)	$3\frac{3}{8}$
d. $\frac{5}{2}$	$\frac{4}{2} + \frac{1}{2} = 2 + \frac{1}{2}$	$2\frac{1}{2}$
e. $\frac{31}{16}$	$\frac{16}{16} + \frac{15}{16} = 1 + \frac{15}{16}$ $\left(\frac{31 - 16}{16}\right)$	$1\frac{15}{16}$
f. $\frac{19}{6}$	$\frac{18}{6} + \frac{1}{6} = 3 + \frac{1}{6}$	$3\frac{1}{6}$

Answers to Self-Correcting Exercises on Page 27 of this Lesson

1. (a) $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ (Prime factors of 8 are 2, 2, 2 and of 12 are 2, 2, and 3. H.C.F. is 4.)

(b) $\frac{20}{60} = \frac{20 \div 20}{60 \div 20} = \frac{1}{3}$

(c) $\frac{50}{75} = \frac{50 \div 25}{75 \div 25} = \frac{2}{3}$ ($50 = 2 \times 5 \times 5$, $75 = 3 \times 5 \times 5$, H.C.F. $= 5 \times 5 = 25$)

(d) $\frac{16}{20} = \frac{16 \div 4}{20 \div 4} = \frac{4}{5}$ ($16 = 2 \times 2 \times 2 \times 2$, $20 = 2 \times 2 \times 5$, H.C.F. $= 2 \times 2 = 4$)

(e) $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$

(f) $\frac{9}{36} = \frac{9 \div 9}{36 \div 9} = \frac{1}{4}$

(g) $\frac{30}{48} = \frac{30 \div 6}{48 \div 6} = \underline{\frac{5}{8}}$ $\left(30 = \underline{2} \times \underline{3} \times 5, \quad 48 = \underline{2} \times 2 \times 2 \times 2 \times \underline{3}, \right.$ $\left. \text{H.C.F.} = 6 \right)$

(h) $\frac{15}{70} = \frac{15 \div 5}{70 \div 5} = \underline{\frac{3}{14}}$

(i) $\frac{9}{21} = \frac{9 \div 3}{21 \div 3} = \underline{\frac{3}{7}}$

(j) $\frac{25}{75} = \frac{25 \div 25}{75 \div 25} = \underline{\frac{1}{3}}$ $(25 = \underline{5} \times \underline{5}, \quad 75 = 3 \times \underline{5} \times \underline{5}, \quad \text{H.C.F.} = 25)$

End of Lesson 2

Practical Mathematics
Module 2
Lesson 3

OPERATIONS INVOLVING FRACTIONS (I)

Topic One: Adding and Subtracting Like Fractions

A. Like Fractions

> LIKE FRACTIONS are fractions which have the same denominator.
>
> e.g. $\frac{1}{4}$, $\frac{2}{4}$, $\frac{5}{4}$ and $\frac{11}{4}$ are like fractions.

EXERCISES:

1. For each of the following fractions, give three more like fractions.

(a) $\frac{1}{8}$ _____ (d) $\frac{5}{6}$ _____

(b) $\frac{2}{7}$ _____ (e) $\frac{1}{2}$ _____

(c) $\frac{7}{3}$ _____ (f) $\frac{9}{16}$ _____

2. Change each of the following pairs of fractions to like fractions using the denominators given. (For each fraction you are finding an equivalent fraction.)

EXAMPLE 1: $\frac{1}{2}$ and $\frac{1}{3}$ to fractions with denominator 6.

$\frac{1}{2}$ may be changed to 6ths by multiplying both terms by 3.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$\frac{1}{3}$ may be changed to 6ths by multiplying both terms by 2.

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$\frac{1}{2}$ and $\frac{1}{3}$ as like fractions are $\frac{3}{6}$ and $\frac{2}{6}$.

EXAMPLE 2: $\frac{3}{8}$ and $\frac{5}{6}$ to fractions with denominator 24.

$\frac{3}{8}$; To find the number to multiply by, divide 24 by 8.

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$\frac{5}{6}$; To find the number to multiply by, divide 24 by 6.

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

$\frac{3}{8}$ and $\frac{5}{6}$ as like fractions are $\frac{9}{24}$ and $\frac{20}{24}$.

To change fractions to like fractions (fractions with a common denominator), for each fraction:

(i) divide the common denominator by the denominator of that fraction

(ii) multiply both terms of that fraction by the number found.

(a) $\frac{1}{6}$ and $\frac{1}{4}$ to 12ths

$\frac{1}{6} = \frac{1 \times 2}{6 \times 2} = \frac{2}{12}$ (12 was divided by 6 to find 2)

$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$ (12 ÷ 4 = 3; multiply both terms by 3.)

(b) $\frac{1}{3}$ and $\frac{2}{4}$ to 12ths

$\frac{1}{3} = _____ = _____$

$\frac{2}{4} = _____ = _____$

(c) $\frac{1}{4}$ and $\frac{5}{8}$ to 16ths

$\frac{1}{4} = _____ = _____$

$\frac{5}{8} = _____ = _____$

(d) $\frac{5}{6}$ and $\frac{2}{3}$ to 6ths

$\frac{5}{6}$ = _____ = _____

$\frac{2}{3}$ = _____ = _____

(e) $\frac{4}{7}$ and $\frac{1}{3}$ to 21sts

$\frac{4}{7}$ = _____ = _____

$\frac{1}{3}$ = _____ = _____

(f) $\frac{1}{5}$ and $\frac{7}{10}$ to 10ths

$\frac{1}{5}$ = _____ = _____

$\frac{7}{10}$ = _____ = _____

(g) $\frac{2}{3}$ and $\frac{4}{5}$ to 15ths

$\frac{2}{3}$ = _____ = _____

$\frac{4}{5}$ = _____ = _____

(h) $\frac{9}{4}$ and $\frac{2}{5}$ to 20ths

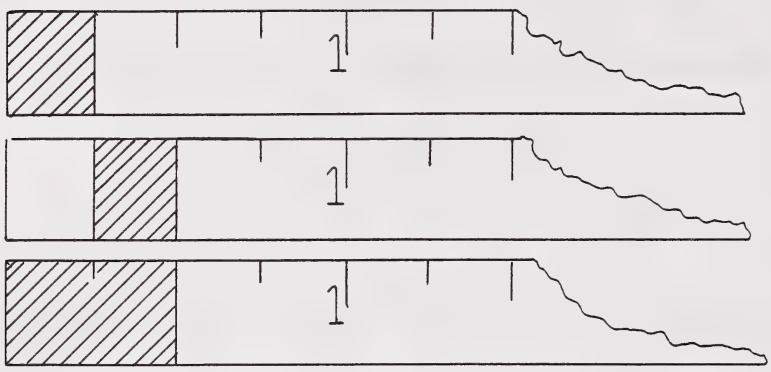
$\frac{9}{4}$ = _____ = _____

$\frac{2}{5}$ = _____ = _____

B. Addition of Like Fractions

I. Addition of Proper and Improper Fractions (Common Fractions)

EXAMPLE 1:

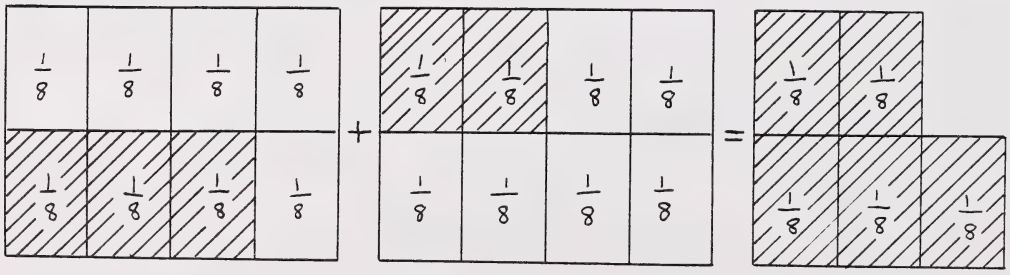


$\frac{1}{4}$ inch + $\frac{1}{4}$ inch = $\frac{1}{2}$ inch

Combine the shaded parts of the ruler (two $\frac{1}{4}$ inches). The total part would be $\frac{1}{2}$ inch.

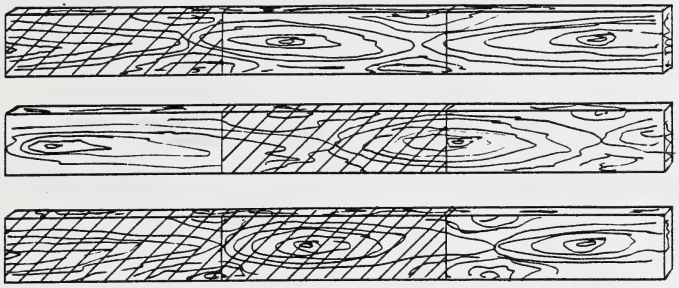
EXAMPLE 2:

Two sheets of plywood are cut into 8ths. Three parts of one sheet are combined with two parts of the other. How many 8ths are combined?



$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

PROBLEM:



A 2" × 4" is cut into thirds. $\frac{1}{3}$ of one board is nailed to one third of the other board. How many thirds result?

$$\frac{1}{3} + \frac{1}{3} = \underline{\qquad\qquad}$$

Consider the above examples. You should be able to see that:

(i) the denominators of the fractions being added are the same.
(ii) only the numerators of the fractions have been added and the result has been put over the common denominator.

i.e. $\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$

$\frac{1}{3} + \frac{1}{3} = \frac{1+1}{3} = \frac{2}{3}$

To add <u>like</u> fractions:

(i) add the numerators
(ii) write the sum over the common denominator
(iii) write the answer in simplest form (or lowest terms) — improper fractions should be renamed as whole numbers or mixed numbers.

The above examples show the addition done horizontally. Addition of fractions may also be done vertically.

e.g. Add $\frac{7}{10}$ and $\frac{1}{10}$

$$\begin{array}{r} \frac{7}{10} \\ + \frac{1}{10} \\ \hline \frac{8}{10} = \frac{4}{5} \end{array}$$

(The numerators of the fractions have been added.)

common denominator

EXERCISES:

1. Add the following fractions. Write your answers in simplest form. Please show your work.
Remember: the denominators of fractions are never added.

(a) $\frac{3}{8} + \frac{1}{8} = \frac{3+1}{8} = \frac{4}{8} = \frac{1}{2}$

(b) $\frac{6}{7} + \frac{5}{7} =$

(c) $\frac{8}{3} + \frac{9}{3} = \frac{+}{3} = \frac{\quad}{3} =$ ← mixed number

(d) $\frac{6}{24} + \frac{6}{24} =$

(e) $\frac{6}{8} + \frac{0}{8} =$

(f) $\frac{18}{24} + \frac{6}{24} =$

(g) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} =$

(h) $\frac{5}{10} + \frac{7}{10} + \frac{2}{10} =$

2. Add the following fractions. Write your answers in simplest form.

(a) $\frac{1}{2}$
$\frac{1}{2}$
$\frac{2}{2} = 1$

(You are adding the numerators of the fractions.)

(b) $\frac{1}{3}$
$\frac{4}{3}$

(c) $\frac{5}{9}$
$\frac{2}{9}$

(d) $\frac{3}{16}$
$\frac{15}{16}$

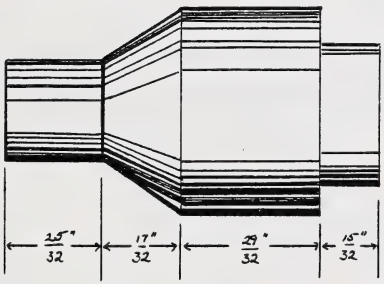
(e) $\frac{2}{8}$
$\frac{3}{8}$
$\frac{1}{8}$

(f) $\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$

(g) $\frac{1}{32}$
$\frac{15}{32}$
$\frac{9}{32}$

(h) $\frac{9}{5}$
$\frac{1}{5}$
$\frac{4}{5}$

3. Find the overall length of this gland. (Express your answer as a mixed number in lowest terms. Be sure to include the unit of measure along with your answer sentence.)



II. Addition of Mixed Numbers

Once you are able to add like fractions, addition of mixed numbers with like fractional parts is quite simple.

> In order to add mixed numbers with <u>like</u> fractional parts:
>
> (i) add the whole number parts
> (ii) add the fractional parts (Express in lowest terms)
> (iii) write the result as a mixed number

EXAMPLE: $1\frac{3}{8} + 3\frac{1}{8}$

$= \underbrace{(1 + 3)}_{\text{sum of whole numbers}} + \underbrace{\frac{3}{8} + \frac{1}{8}}_{\text{sum of fractions}}$

$= 4 + \frac{4}{8}$

$= 4 + \frac{1}{2}$ ← lowest terms

$= 4\frac{1}{2}$ ← mixed number

EXERCISE:

1. Add the following mixed numbers. Give your answers in simplest form.

(a) $3\frac{4}{7} + 5\frac{1}{7} = (3 + 5) + \frac{4}{7} + \frac{1}{7} =$

(b) $2\frac{1}{5} + 2\frac{2}{5} = \left(\quad + \quad\right) + \left(\quad + \quad\right) =$

(c) $7\frac{1}{2} + 3 = (\quad + \quad) + \frac{1}{2} =$

(d) $8\frac{1}{8} + 4\frac{3}{8} =$

(e) $9\frac{7}{12} + 4\frac{1}{12} =$

(f) $3\frac{1}{8} + \frac{5}{8} = 3 + \left(\frac{1}{8} + \quad\right) =$

Sometimes when you add mixed numbers, the sum of the fractional parts is an improper fraction. In such cases you must:

(i) express the improper fraction as a mixed number in lowest terms.
(ii) add the whole-number part of this mixed number to the whole-number sum already obtained.
(iii) write the result as a mixed number.

EXAMPLE 1: $9\frac{7}{8} + 2\frac{3}{8}$

$= (9 + 2) + \left(\frac{7}{8} + \frac{3}{8}\right)$

$= 11 + \left(\frac{10}{8}\right)$ ← improper fraction

$= 11 + \left(1\frac{1}{4}\right)$ ← mixed number in lowest terms

$= \underbrace{(11 + 1)}_{\text{sum of whole numbers}} + \frac{1}{4}$

$= 12\frac{1}{4}$ ← mixed number

If the sum of the fractional parts is a whole number, the result will also be a whole number.

EXAMPLE 2: $3\frac{5}{6} + 4\frac{1}{6}$

$= (3 + 4) + \left(\frac{5}{6} + \frac{1}{6}\right)$

$= 7 + \left(\frac{6}{6}\right)$ ← improper fraction

$= 7 + (1)$ ← whole number

$= 8$ ← whole number

EXERCISES:

1. Add the following mixed numbers. Give your answers in simplest form.

(a) $2\frac{1}{3} + \frac{2}{3} = 2 + \left(\frac{1}{3} + \frac{2}{3}\right) = 2 + \frac{3}{3} =$

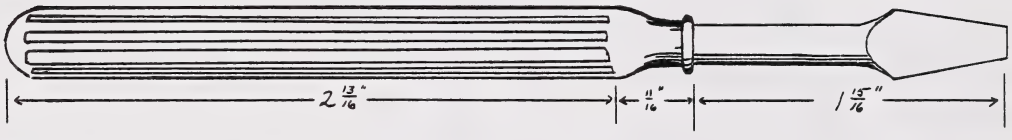
(b) $5\frac{5}{9} + 6\frac{7}{9} = (5 + 6) + \left(\frac{5}{9} + \frac{7}{9}\right) = 11 + \frac{\quad}{9} = ___ + 1\frac{1}{3} = ___\ \frac{1}{3}$

(c) $3\frac{5}{7} + 1\frac{6}{7} = (3 + 1) + \left(\frac{5}{7} + \frac{6}{7}\right) =$

(d) $7\frac{11}{12} + 2\frac{7}{12} = (\quad) + (\quad) =$

(e) $3\frac{3}{10} + 2\frac{7}{10} =$

2. Find the overall length of this screwdriver.



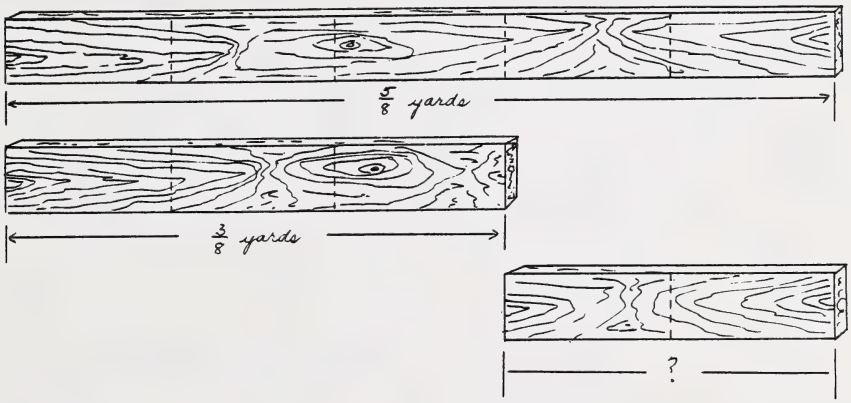
(Express your answer as a mixed number in lowest terms. Be sure to include the unit of measure along with your answer.)

C. Subtraction of Like Fractions

I. Subtraction of Proper and Improper Fractions

EXAMPLE:

You have a board that is $\frac{5}{8}$ yards long. You cut off a piece that is $\frac{3}{8}$ yards long. How long is the piece of board which remains?



From the diagram you should see that the piece which remains is $\frac{2}{8}$ yard long.

$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$$

The steps involved in subtracting like fractions are much the same as those for addition of like fractions. The only difference lies in the fact that you subtract the numerators rather than add them.

> To subtract <u>like</u> fractions:
>
> (i) subtract the numerators
> (ii) write this difference over the common denominator
> (iii) write the answer in simplest form.

EXAMPLE:

$$\frac{19}{24} - \frac{7}{24}$$

$$= \frac{19 - 7}{24}$$ ← difference of numerators; ← common denominator

$$= \frac{12}{24}$$

$$= \frac{1}{2}$$ ← simplest form

EXERCISES:

1. Subtract the following and give the results in simplest form.

(a) $\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

(b) $\frac{5}{8} - \frac{1}{8} = \frac{\quad}{8} = \frac{\quad}{8} = \frac{\quad}{2}$

(c) $\frac{9}{4} - \frac{3}{4} = \quad = \quad$

(d) $\frac{15}{16} - \frac{13}{16} = \quad = \quad$

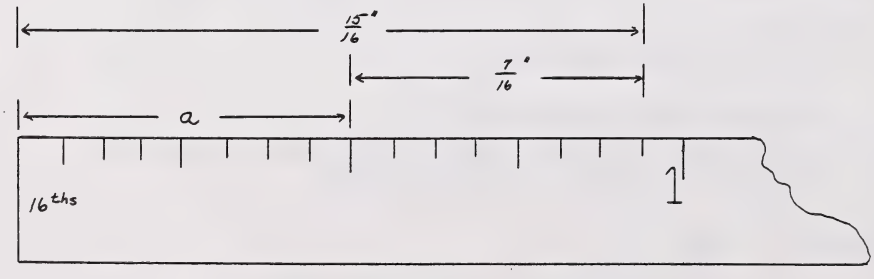
(e) $\frac{7}{8} - \frac{5}{8} = \quad = \quad$

(f) $\frac{3}{2} - \frac{1}{2} = \quad = \quad$

(g) $\frac{5}{6} - \frac{0}{6} = \quad = \quad$

(h) $\frac{31}{32} - \frac{11}{32} = \quad = \quad$

2. For the following ruler diagram, find the measure represented by "a". (Express answer in simplest form and attach the proper units to it.)



(The diagram is 4 times actual size.)

II. Subtraction of Mixed Numbers

In order to subtract mixed numbers with like fractional parts:

(i) subtract their whole-number parts
(ii) subtract their fractional parts (express in lowest terms)
(iii) add the results from parts (i) and (ii).

EXAMPLE: $12\frac{3}{4} - 8\frac{1}{4}$

$= \underbrace{(12 - 8)}_{\text{difference of whole numbers}} \overset{\text{add}}{+} \underbrace{\left(\frac{3}{4} - \frac{1}{4}\right)}_{\text{difference of fractions}}$

$= 4 + \frac{2}{4}$

$= 4 + \frac{1}{2}$ ← lowest terms

$= 4\frac{1}{2}$ ← mixed number

EXERCISE:

1. Subtract the following. Give your answers in simplest form.

(a) $11\frac{5}{8} - 8\frac{3}{8} = (11 - 8) + \left(\frac{5}{8} - \frac{3}{8}\right) = 3 + \frac{2}{8} = 3 + \frac{1}{4} =$ _____

(b) $1\frac{2}{5} - \frac{1}{5} = 1 + \left(\frac{2}{5} - \frac{1}{5}\right) = 1 + \frac{\quad}{5} =$ _____

(c) $9\frac{3}{4} - \frac{1}{4} =$

(d) $7\frac{7}{8} - 3\frac{3}{8} = (7 - 3) + \left(\frac{7}{8} - \frac{3}{8}\right) =$

(e) $105\frac{31}{32} - 9\frac{1}{32} =$

(f) $21\frac{3}{4} - 7\frac{1}{4} =$

(g) $6\frac{3}{8} - 5\frac{3}{8} =$

Sometimes when you subtract mixed numbers, the fractional part of the number you are subtracting is larger than the fractional part of the number you are subtracting from. In such cases, before you subtract, you must rewrite the first mixed number so that its fractional part is an improper fraction. In effect, you borrow 1 from the whole number part and add 1 to the fractional part. Once you have done this, you can go ahead and subtract the mixed numbers as before.

EXAMPLE: $7\frac{1}{8} - 5\frac{3}{8} =$ ▨

Solution

Notice that we cannot subtract $\frac{3}{8}$ from $\frac{1}{8}$ since $\frac{3}{8}$ has a larger numerator than $\frac{1}{8}$.

We must rewrite $7\frac{1}{8}$ as $6\frac{9}{8}$. In other words, we must borrow 1 from the 7 and add $1\left(\text{or } \frac{8}{8}\right)$ to $\frac{1}{8}$.

i.e. $7\frac{1}{8} - 5\frac{3}{8} = 6\frac{9}{8} - 5\frac{3}{8}$

Now we can subtract as before.

$= \underbrace{(6 - 5)}_{\text{difference of whole numbers}} + \underbrace{\left(\frac{9}{8} - \frac{3}{8}\right)}_{\text{difference of fractions}}$

$= 1 + \frac{6}{8}$

$= 1 + \frac{3}{4}$

$= 1\frac{3}{4}$

EXERCISES:

1. Subtract the following. All answers should be in simplest form. (Remember to borrow one when necessary.)

(a) $21\frac{1}{4} - 7\frac{3}{4}$

Borrow 1 from 21 and add $\frac{4}{4}$ to $\frac{1}{4}$.

$= 20\frac{5}{4} - 7\frac{3}{4}$

$= (20 - 7) + \left(\frac{5}{4} - \frac{3}{4}\right)$

$=$

(b) $6\frac{3}{8} - 5\frac{5}{8}$

Borrow 1 from 6 and add $\frac{8}{8}$ to $\frac{3}{8}$.

$= 5\frac{\ }{8} - 5\frac{5}{8}$

$=$

(c) $15 - 14\frac{7}{8}$

$= 14\frac{}{8} - 14\frac{7}{8}$

$=$

(d) $15\frac{7}{8} - 14$

$= (15 - 14) + \frac{7}{8}$

$=$

(e) $2\frac{5}{9} - 2\frac{4}{9}$

$=$

(f) $1\frac{1}{5} - \frac{4}{5}$

$=$

2. A drum of grease, including the container, weighs $420\frac{3}{4}$ pounds. The empty drum weighs $45\frac{1}{4}$ pounds. Find the weight of the grease.

3. The area of a triangle is $27\frac{3}{8}$ square units. The area of a rectangle is $15\frac{7}{8}$ square units. How much smaller is the rectangle than the triangle?

Topic Two: Adding and Subtracting Unlike Fractions

A. Unlike Fractions

Unlike fractions are fractions with different denominators.

e.g. $\frac{1}{2}$, $\frac{9}{7}$, $\frac{5}{6}$, $\frac{1}{4}$ and $\frac{1}{5}$ are unlike fractions.

EXERCISE:

For each of the following pairs of fractions, tell whether they are like or unlike fractions.

(a) $\frac{1}{5}$ and $\frac{1}{8}$ unlike

(b) $\frac{5}{2}$ and $\frac{8}{2}$ _____

(c) $\frac{3}{4}$ and $\frac{1}{4}$ _____

(d) $\frac{1}{5}$ and $\frac{6}{5}$ _____

(e) $\frac{3}{4}$ and $\frac{3}{5}$ _____

(f) $\frac{63}{16}$ and $\frac{32}{8}$ _____

> Unlike fractions can not be added or subtracted until they are expressed as like fractions.

B. Lowest Common Denominator

(a) To add $\frac{1}{2}$ and $\frac{2}{3}$ we need a common denominator.

Some fractions equivalent to $\frac{1}{2}$ are $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and $\frac{6}{12}$.

Equivalent fractions of $\frac{2}{3}$ are $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$.

From the lists of equivalent fractions we should be able to see two pairs of like fractions.

The like fractions are: _____ and _____

_____ and _____

The denominator 6 is called the LOWEST COMMON DENOMINATOR of 2 and 3.

EXERCISE: For each pair of fractions find a pair of like fractions. The common denominator should be the lowest common denominator of the denominators.

(a) $\frac{1}{2}$ and $\frac{1}{3}$

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

$\frac{1}{3} = \frac{2}{6} =$

$\frac{3}{6}$ and $\frac{2}{6}$ are like fractions

Follow this example.

(b) $\frac{3}{4}$ and $\frac{5}{6}$

$\frac{3}{4} = \frac{6}{8} = \frac{\quad}{12}$

$\frac{5}{6} = \frac{\quad}{12}$

$\frac{\quad}{12}$ and $\frac{\quad}{12}$ are like fractions.

(c) $\frac{1}{3}$ and $\frac{2}{9}$

(d) $\frac{1}{4}$ and $\frac{2}{5}$

(e) $\frac{1}{3}$ and $\frac{4}{5}$

I. Multiples

To be a common denominator, a number must be a multiple of each of the denominators.

> A MULTIPLE of a given number is a number that can be divided evenly by the given number.
>
> e.g. 12 is a multiple of 3 (3 divides evenly into 12)

EXERCISES:

1. For each of the numbers below, give two multiples of the number.

   (a) 2; 12, 16 (e) 4; _____, _____

   (b) 7; _____, _____ (f) 9; _____, _____

   (c) 5; _____, _____ (g) 11; _____, _____

   (d) 8; _____, _____ (h) 25; _____, _____

2. Below are pairs of numbers. Write "yes" after each pair in which the second number is a multiple of the first. (Hint: See if the first number divides evenly into the second.)

   (a) 15, 120 yes (f) 7, 54 _____

   (b) 3, 81 _____ (g) 11, 99 _____

   (c) 9, 18 _____ (h) 25, 605 _____

   (d) 7, 24 _____ (i) 16, 64 _____

   (e) 8, 56 _____ (j) 10, 10,000 _____

II. Least Common Multiple

To find the lowest common denominator (LCD), we must find the least common multiple of the denominators.

> The LEAST COMMON MULTIPLE (LCM) of given numbers is the smallest multiple of each of the given numbers.

Given 5, 3 and 15. Are 15 and 30 both multiples of these numbers?

_____

Is 15 or 30 the least common multiple? _____

To find the least common multiple of numbers, we use a method which involves finding the prime factors of the numbers.

> REMEMBER! A PRIME NUMBER is a number which can only be divided evenly by itself and 1.
>
> 20 is not a prime because it is divisible by 2, 4, 5 and 10.
> 23 is a prime since it can only be divided by 1 and itself.
>
> What is 29 divisible by? _____
>
> Is 29 a prime? _____
>
> (Remember that 1 is not considered to be a prime.)

The L.C.M. will be the number which contains all of the different prime factors found in the given numbers.

1. Find the prime factors of each number.
2. Use each prime factor the greatest number of times is appears in any one of the numbers.

EXAMPLE 1: Find the L.C.M. of 15, 21 and 35.

Solution

$15 = 3 \times 5$
$21 = 3 \times 7$
$35 = 5 \times 7$

Each number is factored into its prime factors. Note that each prime is used only once in each number so must appear only once in the L.C.M.

The different primes are 3, 5 and 7.
The LCM is $3 \times 5 \times 7$ or 105.

PROBLEM: Find the LCM of 10, 15, and 6.

Solution

10 = _____ × _____
15 = _____ × _____
6 = _____ × _____

Find the prime factors.

L.C.M. = _____ × _____ × _____ = _____ ← Find the product of the different primes.

EXAMPLE 2: Find the LCM of 8, 12 and 4.

Solution

$8 = 2 \times 2 \times 2$ ← 2 is used 3 times

$12 = 2 \times 2 \times 3$ ← 2 is used twice - 3 is used once

$4 = 2 \times 2$ ← 2 is used twice

The different factors are 2 and 3. What is the most number of times each factor appears?
2 appears 3 times in 8. 3 appears only once.
The LCM is $\underbrace{2 \times 2 \times 2}_{\text{3 factors}} \times 3$ or 24.

PROBLEM: Find the LCD of $\frac{1}{4}$, $\frac{2}{6}$ and $\frac{5}{8}$. (You must find the LCM of 4, 6 and 8.)

Solution

4 = _____ × _____

6 = _____ × _____

8 = _____ × _____ × _____

L.C.M. = _____ × _____ × _____ × _____ = _____

The L.C.D. of $\frac{1}{4}$, $\frac{2}{6}$, and $\frac{5}{8}$ is _____.

EXERCISE: Find the LCM for each of the following sets of numbers. The first one has been done for you. This is a self-correcting exercise. Answers can be found on page 35 of this lesson.

Each factor you list must be divisible only by itself and 1.

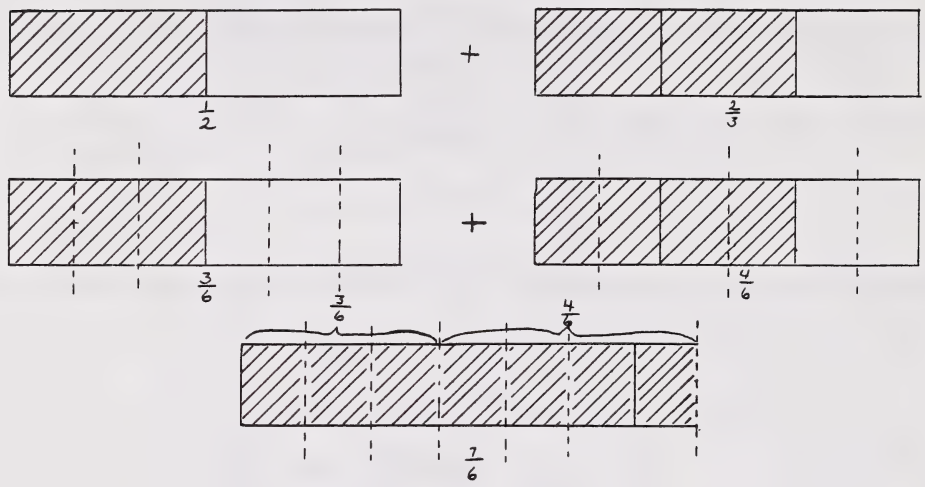
Number	Number Expressed in Prime Factors	No. of Times Each Prime Factor is Used	L.C.M.
1. 9 18 27	3×3 $2 \times 3 \times 3$ $3 \times 3 \times 3$	3 is used twice 2 is used once 3 is used twice 3 is used three times	L.C.M. = 3 (three times) × 2 (Once) = $3 \times 3 \times 3 \times 2$ = 54
2. 3 8 12	prime number __ × __ × __ __ × __ × __	3 is used once. __ is used _____ __ is used _____ __ is used _____	L.C.M. = 3 ( ) × 2 ( ) = __×__×__×__ = ____

Number	Number Expressed in Prime Factors	No. of Times Each Prime Factor is Used	L.C.M.
3. 8	2 × 2 × 2	2 is used 3 times	L.C.M. = 2 (___ times)
9	___ × ___	__ is used _____	× _____
36	__×__×__×__	__ is used _____ __ is used _____	=
4. 21	__ × __	__ is used _____ __ is used _____	
77	__ × __	__ is used _____ __ is used _____	
5. 33		__ is used _____ __ is used _____	
16		__ is used _____	
11	prime number	__ is used _____	
6. 2		__ is used _____	
5		__ is used _____	
10		__ is used _____ __ is used _____	
7. 10		__ is used _____ __ is used _____	
15		__ is used _____ __ is used _____	
6		__ is used _____ __ is used _____	

C. Addition of Unlike Fractions

I. Addition of Proper and Improper Fractions (with different denominators)

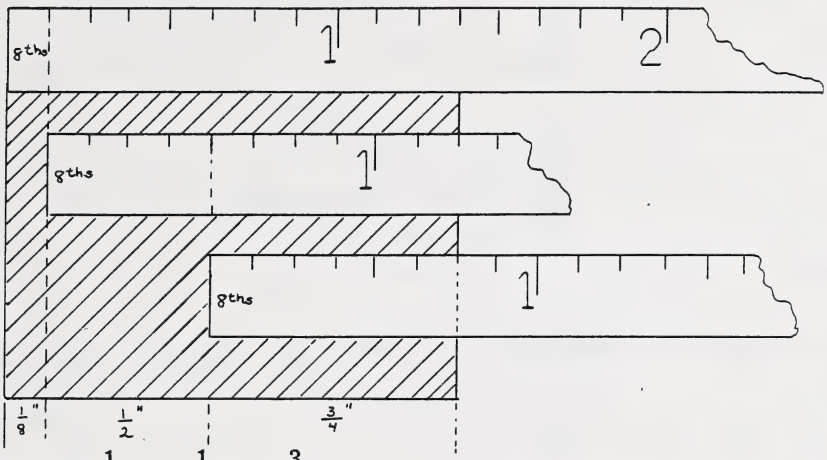
EXAMPLE: Add $\frac{1}{2}$ and $\frac{2}{3}$.



From the diagram, if the rectangles are divided up so that each has 6 equal parts, we see $\frac{1}{2}$ is equal to $\frac{3}{6}$ and $\frac{2}{3}$ is equal to $\frac{4}{6}$. Counting each shaded division we find there are 7 parts, each of which represents $\frac{1}{6}$.

Thus, $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

PROBLEM: In the drawing below, find the overall length of the shaded portion.



Solution: $\frac{1}{8} + \frac{1}{2} + \frac{3}{4}$

$= \frac{\quad}{8} + \frac{\quad}{8} + \frac{\quad}{8}$

$= \frac{\quad}{8}$

To add unlike fractions:

(i) find the lowest common denominator (L.C.D.) of the fractions
(ii) express each fraction as an equivalent fraction using this common denominator
(iii) add the numerators – put this sum over the common denominator
(iv) express the answer in simplest form

* Only fractions which have a common denominator can be added.

EXAMPLE: $\frac{1}{6} + \frac{3}{8} =$

Solution

The L.C.D. of $\frac{1}{6}$ and $\frac{3}{8}$ is 24. The two fractions must be expressed with denominators of 24.

$\frac{1}{6} + \frac{3}{8}$

$= \frac{4}{24} + \frac{9}{24}$ ← $\frac{1}{6} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}$; $\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$

$= \frac{13}{24}$

Remember that addition and subtraction may also be done vertically. (Generally you can choose which format you prefer.)

$$\begin{array}{rcl} \frac{1}{6} & = & \frac{4}{24} \leftarrow \text{L.C.D.} \\ +\ \frac{3}{8} & = & \frac{9}{24} \leftarrow \text{L.C.D} \\ \hline & & \frac{13}{24} \leftarrow \text{sum of numerators} \\ & & \text{L.C.D.} \end{array}$$

EXERCISES:

1. Find the missing numerators and then add. Express answers in simplest form.

(a) $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} =$

(b) $\frac{1}{4} + \frac{3}{8} = \frac{\quad}{8} + \frac{\quad}{8} =$

(c) $\frac{5}{12} + \frac{1}{4} = \frac{\quad}{12} + \frac{\quad}{12} =$

(d) $\frac{1}{8} + \frac{1}{6} = \frac{\quad}{24} + \frac{\quad}{24} =$

(e) $\frac{5}{6} = \frac{\quad}{18}$

$\frac{1}{9} = \frac{\quad}{18}$

(f) $\frac{3}{10} = \frac{\quad}{20}$

$\frac{1}{4} = \frac{\quad}{20}$

2. Simplify by adding. Show all work. Express all answers in simplest form. The answers to these exercises can be found on page 36 of this lesson.

(a) $\frac{4}{5} + \frac{7}{8} = \left(\frac{4 \times 8}{5 \times 8}\right) + \left(\frac{7 \times 5}{8 \times 5}\right) = \frac{32}{40} + \frac{35}{40} = \frac{67}{40} = 1\frac{27}{40}$

(b) $\frac{1}{2} + \frac{2}{3} = \left(\frac{1 \times 3}{2 \times 3}\right) + \left(\frac{2 \times 2}{3 \times 2}\right) =$

(c) $\frac{3}{4} + \frac{2}{5} =$

(d) $\frac{4}{5} + \frac{3}{5} =$

(e) $\frac{8}{5} + \frac{9}{11} =$

(f) $\frac{5}{6} + \frac{2}{15} =$

(g) $\frac{2}{3} = \frac{\quad}{60}$

$\frac{3}{4} = \frac{\quad}{60}$

$\frac{4}{5} = \frac{\quad}{60}$

(h) $\frac{5}{8} =$

$\frac{1}{6} =$

$\frac{7}{12} =$ _____

(i) $\frac{5}{6} =$

$\frac{9}{7} =$

$\frac{17}{14} =$ _____

(j) $\frac{7}{16} =$

$\frac{5}{8} =$

$\frac{3}{4} =$ _____

3. Find the following sums.

(a) $\frac{3}{7} = \frac{15}{35}$

$\frac{1}{5} = \frac{7}{35}$

(b) $\frac{1}{8} =$

$\frac{5}{6} =$ _____

(c) $\frac{1}{2} =$

$\frac{1}{9} =$ _____

(d) $\frac{2}{5} =$

$\frac{11}{20} =$ _____

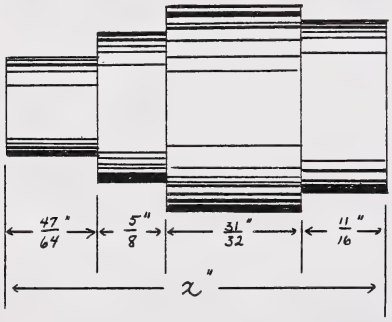
(e) $\frac{72}{5} + \frac{7}{10} = \frac{72 \times 2}{5 \times 2} + \frac{7}{10} =$

(f) $\frac{11}{36} + \frac{5}{24} =$

(g) $\frac{5}{9} + \frac{3}{8} + \frac{2}{15} =$

4. Mr. James spends $\frac{1}{4}$ of his salary for rent, $\frac{1}{3}$ for food, $\frac{1}{6}$ for clothing, and $\frac{1}{8}$ for auto expenses. The remainder of his salary is left for personal expenses and savings. What fraction of his salary is left for personal expenses and savings?

5. In the drawing below, find the overall length indicated by "x". (Be sure to include the units of measure along with your answer.)



II. Addition of Mixed Numbers

You have already taken addition of mixed numbers with like fractional parts and addition of unlike fractions. It should now be very simple to add mixed numbers with unlike fractional parts.

> To add mixed numbers with unlike fractional parts:
> (i) find the L.C.D. of the fractional parts
> (ii) express the fractional parts using this L.C.D.
> (iii) add the whole-number parts
> (iv) add the fractional parts
> (v) write the sum as a mixed number in lowest terms

EXAMPLE: $2\frac{5}{6} + 3\frac{3}{4}$

$= 2\frac{10}{12} + 3\frac{9}{12}$ } The fractional parts, $\frac{5}{6}$ and $\frac{3}{4}$, are expressed to the L.C.D., 12.

$= \underbrace{(2 + 3)}_{\text{sum of whole numbers}} + \underbrace{\left(\frac{10}{12} + \frac{9}{12}\right)}_{\text{sum of fractional parts}}$

$= 5 + \frac{19}{12}$

$= 5 + 1\frac{7}{12}$

$= 6\frac{7}{12}$ ← mixed number in lowest terms

This example could also have been done vertically.

$$\begin{array}{rcl} 2\frac{5}{6} & = & 2\frac{10}{12} \leftarrow \text{L.C.D.} \\ +\ 3\frac{3}{4} & = & 3\frac{9}{12} \leftarrow \text{L.C.D.} \\ \hline \text{sum of whole numbers} \rightarrow 5\frac{19}{12} & = & 6\frac{7}{12} \end{array}$$

sum of fractions

EXERCISES:

1. Do the following addition problems. Express the answers in simplest form.

(a) $6\frac{4}{5} + 3\frac{1}{2} = 6\frac{8}{10} + 3\frac{5}{10} = 9\frac{13}{10} = 9 + 1\frac{3}{10} = 10\frac{3}{10}$

(b) $4\frac{1}{3} + 7\frac{2}{3} =$

(c) $2\frac{1}{4} + 5\frac{5}{6} = 2\frac{3}{12} + 5\frac{\quad}{12} =$

(d) $1\frac{11}{12} + 8\frac{7}{20} =$

(e) $2\frac{1}{2} + 3\frac{1}{11} =$

(f) $15\frac{2}{3} + 23\frac{4}{7} + 35\frac{2}{5} =$

2. Find the following sums. Express answers in simplest form.

(a) $4\frac{5}{6} =$
$+ 2\frac{3}{8} =$ _____

(b) $1\frac{3}{4} =$
$+ 9\frac{7}{12} =$ _____

3. A laminated beam is made of five pieces of lumber. Two of the pieces are each $1\frac{1}{2}$ inches thick, two others are each $2\frac{5}{8}$ inches thick, and the other piece is $3\frac{3}{4}$ inches thick. How thick is the laminated beam?

4. At the end of each hour for 5 hours a racing car was noted to have travelled $117\frac{5}{6}$, $120\frac{7}{8}$, $105\frac{5}{16}$, $110\frac{2}{3}$, and 115 miles. What was the total distance travelled in 5 hours?

D. Subtraction of Unlike Fractions

I. Subtraction of Proper and Improper Fractions

From your knowledge of subtraction of like fractions and changing unlike fractions to like fractions, you should easily be able to subtract unlike fractions.

To subtract unlike fractions:
(i) find the L.C.D. of the fractions — find equivalent fractions using the L.C.D.
(ii) subtract the numerators
(iii) write the result over the common denominator
(iv) express the answer in simplest form

EXAMPLE: $\frac{2}{3} - \frac{7}{11}$

$= \frac{22}{33} - \frac{21}{33}$ } The fractions have been expressed to the L.C.D., 33.

$= \frac{22 - 21}{33}$ ← difference of numerators; ← common denominator

$= \frac{1}{33}$ ← simplest form

Subtractions can also be done vertically.

EXAMPLE:

$$\begin{array}{rcl} \frac{2}{3} & = & \frac{22}{33} \leftarrow \text{L.C.D.} \\ -\ \frac{7}{11} & = & \frac{21}{33} \leftarrow \text{L.C.D.} \\ \hline & & \frac{1}{33} \end{array}$$

$\frac{1}{33}$ ← difference of numerators; ← L.C.D.

EXERCISES:

1. Subtract the following and give the results in simplest form.

(a) $\frac{2}{3} - \frac{1}{3} =$

(b) $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{(3 \times 2)}{8} = \frac{7}{8} - \frac{6}{8} =$

(c) $\frac{11}{16} - \frac{1}{2} =$

(d) $\frac{2}{3} - \frac{1}{6} =$

(e) $\frac{9}{10} - \frac{3}{5} =$

(f) $\frac{8}{9} - \frac{1}{2} =$

(g) $\frac{9}{10} = \frac{81}{90}$

$-\frac{5}{9} = \frac{50}{90}$

$\frac{}{90}$

(h) $\frac{6}{5} =$

$-\frac{3}{7} =$ _____

(i) $\frac{14}{5} =$

$-\frac{7}{15} =$ _____

(j) $\frac{9}{11} =$

$-\frac{1}{6} =$ _____

2. A bond is to be paid as follows: $\frac{1}{3}$ the first year, $\frac{3}{5}$ the second year, and the remainder the third year. How much was paid the third year?

3. A tank full of water has two pipes opening from it. One will empty $\frac{1}{4}$ of the water in the tank in 1 hour and the other $\frac{7}{12}$ of it. What part will both pipes empty in 1 hour?

II. Subtraction of Mixed Numbers

To subtract mixed numbers with unlike fractional parts:
(i) find the L.C.D. of the fractional parts
(ii) express the fractional parts using this L.C.D.
(iii) check to see if the fractional parts can be subtracted (if not you must borrow)
(iv) subtract the whole-number parts
(v) subtract the fractional parts (express in lowest terms)
(vi) add the results from parts (iv) and (v).

EXAMPLE: $9\frac{2}{3} - 6\frac{7}{8}$

$= 9\frac{16}{24} - 6\frac{21}{24}$

We cannot subtract $\frac{21}{24}$ from $\frac{16}{24}$. We must rewrite $9\frac{16}{24}$ as $8\frac{40}{24}$.

$= 8\frac{40}{24} - 6\frac{21}{24}$

Subtract as before.

$= \underbrace{(8 - 6)}_{\text{difference of whole numbers}} + \underbrace{\left(\frac{40}{24} - \frac{21}{24}\right)}_{\text{difference of fractions}}$

$= 2 + \frac{19}{24}$

$= 2\frac{19}{24}$ ← mixed number in lowest terms

Mixed numbers can also be subtracted vertically.

EXAMPLE:

Take fractions to L.C.D.

$$\begin{array}{rcccl} 9\frac{2}{3} & = & \left(9\frac{16}{24}\right) & = & \left(8\frac{40}{24}\right) \leftarrow \text{Borrowing step} \\ -\,6\frac{7}{8} & = & 6\frac{21}{24} & = & 6\frac{21}{24} \\ & & & & \text{difference of numerators} \\ & & & & 2\frac{19}{24} \leftarrow \text{L.C.D.} \end{array}$$

difference of whole numbers

EXERCISES:

1. Simplify the following.

(a) $6\frac{1}{4} - 5\frac{1}{3} = 6\frac{3}{12} - 5\frac{4}{12} = 5\frac{15}{12} - 5\frac{4}{12} = \frac{11}{12}$

(b) $9\frac{4}{5} - 3\frac{13}{15} = 9\frac{\quad}{15} - 3\frac{13}{15} = 8\frac{\quad}{15} - 3\frac{13}{15} = 5\frac{\quad}{15}$

(c) $3\frac{4}{5} - 1\frac{2}{3} =$

(d) $7\frac{3}{5} - \frac{3}{4} =$

(e) $15\frac{1}{6} - \frac{3}{5} =$

(f) $22\frac{3}{4} - 11\frac{5}{6} =$

(g) $10\frac{1}{3} - 4\frac{2}{5} =$

(h) $9 - \frac{3}{13} = 8\frac{\quad}{13} - \frac{3}{13} =$

(i) $5 - \frac{1}{5} =$

(j) $5\frac{7}{12} - \frac{13}{15} =$

2. Subtract. Write answers in simplest form.

(a) $3\frac{1}{4} = \quad = \qquad - 1\frac{5}{8} = ____ = ____$

(b) $8\frac{5}{12} = \quad = \qquad - 2\frac{7}{8} = ____ = ____$

3. A rough casting weighs $8\frac{5}{16}$ lb. When it is finished it weighs $7\frac{5}{8}$ lb. How much material was machined off?

4. From a 16-ft. board, a carpenter cuts the following lengths: $2\frac{1}{2}$ ft., $3\frac{1}{4}$ ft., $1\frac{7}{16}$ ft., and $1\frac{5}{8}$ ft. How many feet of board are left? (Hint: First, find the total length cut off.)

5. The distance from outside to outside between two holes in a steel plate is $6\frac{2}{3}$ inches. If one hole is $1\frac{1}{6}$ inches in diameter and the other is $2\frac{1}{24}$ inches in diameter, find the length of metal between the holes. (Draw yourself a diagram.)

6. A contractor started construction of a stretch of highway of $15\frac{3}{8}$ miles in length in May. By September three sections were completed that were respectively $3\frac{1}{2}$ miles, $5\frac{3}{4}$ miles, and $2\frac{3}{16}$ miles in length. How many miles of highway remained uncompleted?

Answers to Self-Correcting Exercises on Page 20 of this Lesson

Number	Number Expressed in Prime Factors	No. of Times Each Prime Factor is Used	L.C.M.
2. 3 8 12	prime number $2 \times 2 \times 2$ $2 \times 2 \times 3$	3 is used once. 2 is used 3 times 2 is used twice 3 is used once	L.C.M. = 3 (once) × 2 (3 times) = $3 \times 2 \times 2 \times 2$ = 24
3. 8 9 36	$2 \times 2 \times 2$ 3×3 $2 \times 2 \times 3 \times 3$	2 is used 3 times 3 is used twice 2 is used twice 3 is used twice	L.C.M. = 2 (3 times) × 3 (2 times) = $2 \times 2 \times 2 \times 3 \times 3$ = 72 (A factor is only considered the greatest no. of times it is used in any one of the numbers)
4. 21 77	3×7 7×11	3 is used once 7 is used once 7 is used once 11 is used once	L.C.M. = 3 (once) × 7 (once) × 11 (once) = $3 \times 7 \times 11$ = 231
5. 33 16 11	3×11 $2 \times 2 \times 2 \times 2$ prime number	3 is used once 11 is used once 2 is used 4 times 11 is used once	L.C.M. = 3 (once) × 11 (once) × 2 (4 times) = $3 \times 11 \times 2 \times 2 \times 2 \times 2$ = 528
6. 2 5 10	prime number prime number 2×5	2 is used once 5 is used once 2 is used once 5 is used once	L.C.M. = 2 (once) × 5 (once) = 2×5 = 10
7. 10 15 6	2×5 3×5 2×3	2 is used once 5 is used once 3 is used once 5 is used once 2 is used once 3 is used once	L.C.M. = 2 (once) × 3 (once) × 5 (once) = $2 \times 3 \times 5$ = 30

Answers to Self-Correcting Exercises on Page 24 of this Lesson

2. (b) $\frac{1}{2} + \frac{2}{3} = \underbrace{\frac{1 \times 3}{2 \times 3} + \frac{2 \times 2}{3 \times 2}}_{\text{L.C.D. is 6}} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = \boxed{1\frac{1}{6}}$

(c) $\frac{3}{4} + \frac{2}{5} = \frac{3 \times 5}{4 \times 5} + \frac{2 \times 4}{5 \times 4} = \frac{15}{20} + \frac{8}{20} = \underbrace{\frac{23}{20}}_{\text{improper fraction}} = \boxed{1\frac{3}{20}}$ ⟶ mixed number

(d) $\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = \boxed{1\frac{2}{5}}$

(e) $\frac{8}{5} + \frac{9}{11} = \frac{8 \times 11}{55} + \frac{9 \times 5}{55} = \frac{88}{55} + \frac{45}{55} = \frac{133}{55} = \boxed{2\frac{23}{55}}$ (133 = Sum of 88 and 45)

(f) $\underbrace{\frac{5}{6} + \frac{2}{15}}_{(6 = 2 \times 3;\quad 15 = 3 \times 5;\quad \text{L.C.D.} = 2 \times 3 \times 5 = 30)} = \frac{5 \times 5}{30} + \frac{2 \times 2}{30} = \frac{25}{30} + \frac{4}{30} = \boxed{\frac{29}{30}}$

(g) $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{2 \times 20}{60} + \frac{3 \times 15}{60} + \frac{4 \times 12}{60} = \frac{40 + 45 + 48}{60} = \frac{133}{60} = \boxed{2\frac{13}{60}}$

$(3 = 3 \times 1;\quad 4 = 2 \times 2;\quad 5 = 5 \times 1;\quad \text{L.C.D.} = 2 \times 2 \times 3 \times 5)$

(h) $\frac{5}{8} + \frac{1}{6} + \frac{7}{12} = \frac{5 \times 3}{24} + \frac{1 \times 4}{24} + \frac{7 \times 2}{24} = \underbrace{\frac{15 + 4 + 14}{24}}_{\text{add numerators}} = \frac{33}{24} = 1\frac{9}{24} = \boxed{1\frac{3}{8}}$

(i) $\frac{5}{6} + \frac{9}{7} + \frac{17}{14} = \frac{5 \times 7}{42} + \frac{9 \times 6}{42} + \frac{17 \times 3}{42} = \frac{35 + 54 + 51}{42} = \frac{140}{42} = 3\frac{14}{42} = \boxed{3\frac{1}{3}}$

$(6 = 2 \times 3;\quad 7 = 7 \times 1;\quad 14 = 2 \times 7;\quad \text{L.C.D.} = 2 \times 3 \times 7 = 42)$

(j) $\frac{7}{16} + \frac{5}{8} + \frac{3}{4} = \frac{7}{16} + \frac{10}{16} + \frac{12}{16} = \frac{7 + 10 + 12}{16} = \frac{29}{16} = \boxed{1\frac{13}{16}}$

End of Lesson 3

Practical Mathematics
Module 2
Lesson 4

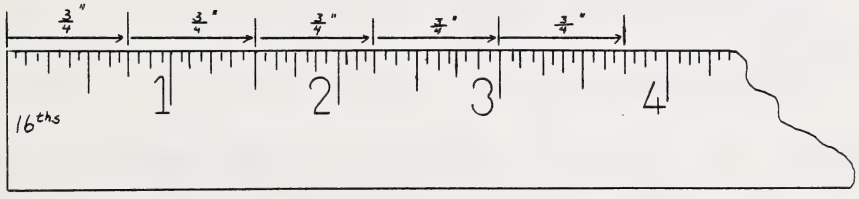
OPERATIONS INVOLVING FRACTIONS (II)

Topic One: Multiplication of Fractions

A. Multiplication of Common Fractions

How many inches is $5 \times \frac{3}{4}$"?

Consider the ruler below. Thinking of multiplication as repeated addition, we could say that $5 \times \frac{3}{4}$" is equivalent to adding $\frac{3}{4}$" to itself 5 times.



From the diagram, we see the result is $\frac{15}{4}$" or $3\frac{3}{4}$".

EXERCISE: With the help of the above diagram, find the following:

(a) $3 \times \frac{1}{4}$" = $\frac{3}{4}$"

(b) $4 \times \frac{3}{4}$" =

(c) $2 \times \frac{3}{8}$" =

(d) $\frac{1}{4}$ of 4" =

(e) $\frac{3}{4}$ of 4" =

(f) $\frac{3}{8}$ of 2" =

(g) $\frac{1}{3} \times 3$" =

(h) $\frac{1}{16} \times 2$" =

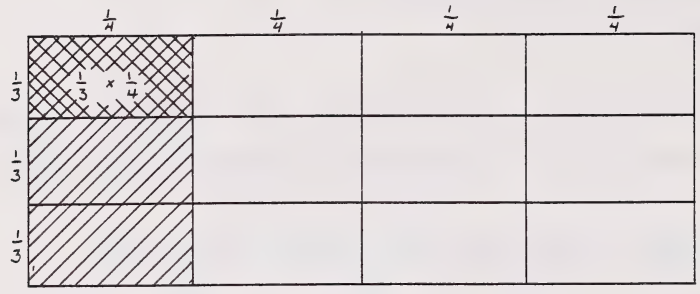
(i) $\frac{1}{8}$ of 3" =

(j) $\frac{5}{8} \times 4$" =

In each of the above we are multiplying a fraction by an integer. Now let's try multiplying a fraction by a fraction.

Multiply: $\frac{1}{3} \times \frac{1}{4}$

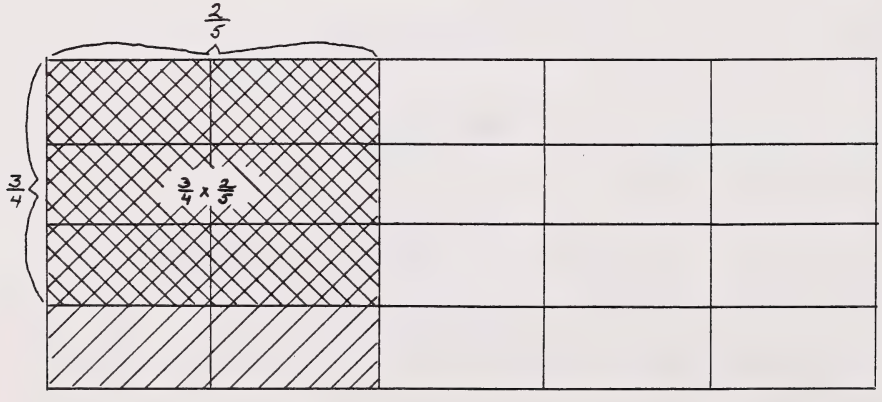
A diagram can be used to illustrate this multiplication.



To multiply $\frac{1}{3} \times \frac{1}{4}$ means to take $\frac{1}{3}$ of $\frac{1}{4}$. In the diagram, the first column of rectangles represents $\frac{1}{4}$. Taking $\frac{1}{3}$ of this, we see that $\frac{1}{3}$ of $\frac{1}{4}$ is equal to 1 of 12 equal parts or $\frac{1}{12}$. (On the diagram, one of the 12 rectangles is cross-hatched.)

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

Multiply: $\frac{3}{4} \times \frac{2}{5}$



The rectangle is first divided into fifths then fourths. We must shade 2 of the fifths. Our product will then be $\frac{3}{4}$ of the 2 fifths. From the diagram we see:

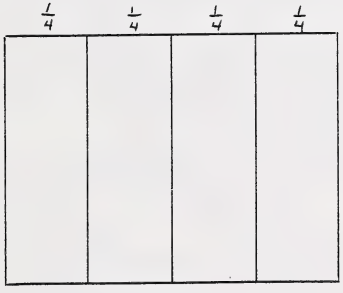
(a) the larger rectangle has been divided into _____ smaller rectangles.

(b) the rectangles are all equal in size.

(c) the number of rectangles in the double-shaded area is _____ rectangles.

(d) the product of $\frac{3}{4} \times \frac{2}{5}$ is _____.

By using the rectangle below, illustrate and find the product $3 \times \frac{1}{4}$.
(Take 3 of $\frac{1}{4}$)



$3 \times \frac{1}{4} =$ _____

* NOTE: $3 \times \frac{1}{4} = \frac{3}{1} \times \frac{1}{4}$, since 3 is equal to $\frac{3}{1}$.

Any whole number is equivalent to a fraction having the whole number as its numerator and 1 as its denominator.

e.g. $0 = \frac{0}{1}$; $4 = \frac{4}{1}$; $9 = \frac{9}{1}$; $100 = \frac{100}{1}$

Study the previous examples to see how fractions are multiplied.

> The product of two fractions is a fraction having for its numerator the product of the numerators of the original fractions and having for its denominator the product of the denominators of the original fractions.
>
> product of fractions = $\frac{\text{product of numerators}}{\text{product of denominators}}$

EXAMPLE 1:

$$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$

(product of numerators; product of denominators)

EXAMPLE 2:

$$\frac{1}{4} \times \frac{9}{5} = \frac{1 \times 9}{4 \times 5} = \frac{9}{20}$$

EXERCISES:

1. Find the following products expressing the answers in simplest form. The answers to this exercise can be found on page 27 of this lesson. All your work should be shown.

(a) $\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7} =$

(b) $\frac{2}{5} \times \frac{1}{3} = \quad =$

(c) $\frac{15}{16} \times \frac{3}{4} = \quad =$

(d) $\frac{3}{5} \times 3 = \quad =$

(e) $\frac{2}{3} \times \frac{4}{5} = \quad =$

(f) $\frac{1}{4} \times \frac{7}{8} = \quad =$

(g) $\frac{1}{8} \times 9 = \quad =$

(h) $\frac{5}{6} \times \frac{1}{4} = \quad =$

(i) $\frac{3}{10} \times \frac{3}{2} = \quad =$

(j) $\frac{10}{3} \times \frac{2}{3} = \quad =$

2. An alloy used for bearings in machinery is $\frac{24}{29}$ copper, $\frac{4}{29}$ tin, and $\frac{1}{29}$ zinc. How many kilograms of each metal are in 157 kilograms of the alloy? Show your calculations.

I. Multiplication of Fractions Using Cancellation

You should recall that to reduce a fraction to lowest terms (i.e. simplest form) you can "divide out" common factors.

e.g. $\frac{36}{44} = \frac{36 \div 4}{44 \div 4} = \frac{9}{11}$

This same result can be obtained by "cancelling" common factors from the numerator and denominator of a fraction.

e.g. $\frac{36}{44} = \frac{9 \times \cancel{4}^{1}}{11 \times \cancel{4}_{1}} = \frac{9}{11}$ *The small numeral 1 tells us that 4 ÷ 4 = 1*

or $\frac{\cancel{36}^{9}}{\cancel{44}_{11}} = \frac{9}{11}$

(4 has been cancelled from both the numerator and the denominator. Since 36 ÷ 4 = 9, 9 is placed above the 36. Since 44 ÷ 4 = 11, 11 is placed below the 44.)

EXERCISE:

Reduce the following fractions to lowest terms by using the cancellation method.

(a) $\frac{9}{12} = \frac{\cancel{3}^{1} \times 3}{\cancel{3}_{1} \times 4} = \frac{3}{4}$

(b) $\frac{8}{12} = \frac{4 \times 2}{4 \times 3} = \frac{\quad}{3}$

(c) $\frac{4}{16} =$

(d) $\frac{5}{20} =$

(e) $\frac{18}{24} =$

(f) $\frac{5}{10} =$

(g) $\frac{7}{21} =$

(h) $\frac{5}{25} =$

(i) $\frac{9}{27} =$

(j) $\frac{16}{24} =$

This same "cancellation" method can be used for simplifying when multiplying fractions.

To cancel, when multiplying, a common factor of both numerator and any denominator may be divided from the fractions. Cancelling before multiplying saves you from having to simplify your final answer.

EXAMPLE 1:

$$\frac{1}{2} \times \frac{2}{7} = \frac{1 \times 2}{2 \times 7}$$

product of numerators

product of denominators

Cancel at this step. The factor 2 can be cancelled from both the numerator and denominator.

$$= \frac{1 \times \cancel{2}^{1}}{\cancel{2}_{1} \times 7}$$

$$= \frac{1 \times 1}{1 \times 7}$$

product of remaining factors in numerator

product of remaining factors in denominator

$$= \frac{1}{7}$$

EXAMPLE 2:

$$\frac{8}{27} \times \frac{9}{20} = \frac{\cancel{8}^{2} \times 9}{27 \times \cancel{20}_{5}}$$

The common factor 4 divides into the numerator 8 and the denominator 20. ($8 \div 4 = 2$; $20 \div 4 = 5$)

$$= \frac{\cancel{8}^{2} \times \cancel{9}^{3}}{\cancel{27}_{3} \times \cancel{20}_{5}}$$

The common factor 9 divides into the numerator 9 and the denominator 27. ($9 \div 9 = 1$; $27 \div 9 = 3$)

$$= \frac{2 \times 1}{3 \times 5}$$

product of remaining factors in numerator

$$= \frac{2}{15}$$

product of remaining factors in denominator

* A common factor can be cancelled from both a numerator and from a denominator when multiplying common fractions. Do not cancel when you are adding or subtracting fractions.

EXERCISES:

1. Simplify the following by using the cancellation method.

(a) $\frac{4}{9} \times \frac{3}{8} = \frac{\cancel{4}^{1} \times \cancel{3}^{1}}{\cancel{9}_{3} \times \cancel{8}_{2}}$

$= \frac{1 \times 1}{3 \times 2}$

$= \frac{1}{6}$

(b) $\frac{5}{12} \times \frac{3}{10} =$

(c) $\frac{6}{5} \times \frac{15}{18} =$

(d) $\frac{3}{8} \times \frac{8}{9} =$

(e) $\frac{5}{6} \times \frac{12}{5} =$

(f) $\frac{3}{4} \times \frac{20}{27} =$

(g) $\frac{6}{9} \times \frac{18}{36} =$

(h) $\frac{5 \times 6}{8 \times 15} =$

(i) $\frac{4}{15} \times \frac{10}{28} =$

(j) $\frac{7}{12} \times \frac{8}{21} =$

(k) $\frac{9}{10} \times \frac{25}{21} =$

(l) $\frac{7}{8} \times \frac{12}{14} =$

2. Find the following products. Make sure your answers are in lowest terms.

(a) $\textcircled{24} \times \frac{5}{24} = \frac{\cancel{24}^{1} \times 5}{1 \times \cancel{24}_{1}}$

$24 = \frac{24}{1}$

$=$

(b) $10 \times \frac{3}{10} =$

(c) $100 \times \frac{3}{100} =$

(d) $100 \times \frac{3}{10} =$

(e) $10 \times \frac{7}{100} =$

(f) $\frac{9}{10} \times \frac{20}{100} =$

(g) $24 \times \frac{5}{6} =$

(h) $\frac{9}{15} \times \frac{7}{9} =$

(i) $\frac{7}{10} \times \frac{100}{21} \times \frac{9}{10} = \frac{7 \times 100 \times 9}{10 \times 21 \times 10}$

(j) $\frac{5}{7} \times 49 \times \frac{2}{5} =$

3. A man in partnership owns $\frac{3}{4}$ of an apartment building which is valued at \$960,000. He gives $\frac{7}{16}$ of his share to his son. What is the value of the gift? (HINT: What is the value of the man's share?)

II. Multiplying by the Multiplicative Inverse (Reciprocal) of a Fraction

EXAMPLE:

$$\frac{2}{5} \times \frac{5}{2} = \frac{\cancel{2}^{1} \times \cancel{5}^{1}}{\cancel{5}_{1} \times \cancel{2}_{1}} = 1$$

$\frac{2}{5}$ and $\frac{5}{2}$ are called RECIPROCALS (or MULTIPLICATIVE INVERSES) of each other.

> When we interchange the numerator and denominator of a fraction, we obtain the RECIPROCAL (or MULTIPLICATIVE INVERSE) of that fraction.

EXAMPLES:

$\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals.

$\frac{1}{6}$ and $\frac{6}{1}$ are reciprocals.

(5) and $\frac{1}{5}$ are reciprocals.

$5 = \frac{5}{1}$

Give the multiplicative inverse of each of the following.

(a) $\frac{1}{8}$ $\frac{8}{1}$

(b) $\frac{2}{7}$ _____

(c) $\frac{5}{4}$ _____

(d) 9 _____

(e) $\frac{10}{3}$ _____

(f) $\frac{31}{100}$ _____

(g) $\frac{11}{7}$ _____

(h) $\frac{1}{2}$ _____

> If a fraction is multiplied by its reciprocal, the product is always 1.

EXAMPLES:

$$\frac{3}{4} \times \frac{4}{3} = \frac{\cancel{3}^{1} \times \cancel{4}^{1}}{\cancel{4}_{1} \times \cancel{3}_{1}} = 1$$

$$\frac{1}{6} \times \frac{6}{1} = \frac{\cancel{1}^{1} \times \cancel{6}^{1}}{\cancel{6}_{1} \times \cancel{1}_{1}} = 1$$

EXERCISES:

1. Find the following products.

(a) $\frac{9}{5} \times \frac{5}{9} = 1$

(b) $\frac{7}{6} \times \frac{6}{7} =$

(c) $\frac{2}{3} \times \frac{3}{2} =$

(d) $9 \times \frac{1}{9} =$

(e) $100 \times \frac{1}{100} =$

(f) $\frac{15}{16} \times \frac{16}{15} =$

2. Fill in the blanks.

(a) $\frac{1}{10} \times$ _____ = 1. The multiplicative inverse of $\frac{1}{10}$ is _____.

(b) $\frac{2}{7} \times$ _____ = 1. The multiplicative inverse of $\frac{2}{7}$ is _____.

(c) $7 \times$ _____ = 1. The multiplicative inverse of 7 is _____.

(d) $\frac{7}{6} \times$ _____ = 1. The multiplicative inverse of $\frac{7}{6}$ is _____.

B. Multiplication of Mixed Numbers

To multiply mixed numbers:
(i) Change the mixed numbers to their equivalent improper fractions.
(ii) Multiply the improper fractions as you would common fractions.
(iii) Express the result in simplest form.

EXAMPLES:

$$3\frac{1}{2} \times 2\frac{2}{3} = \underbrace{\frac{7}{2} \times \frac{8}{3}}_{\text{improper fractions}} = \underbrace{\frac{7 \times \cancel{8}^{4}}{\cancel{2} \times 3} = \frac{7 \times 4}{1 \times 3} = \frac{28}{3}}_{\text{multiply as before}} = 9\frac{1}{3} \leftarrow \text{simplest form}$$

$$9 \times 7\frac{1}{6} = \frac{9}{1} \times \frac{43}{6} = \frac{\cancel{9}^{3} \times 43}{1 \times \cancel{6}_{2}} = \frac{3 \times 43}{1 \times 2} = \frac{129}{2} = 64\frac{1}{2}$$

EXERCISES:

1. Simplify the following.

   The answers to this exercise can be found on page 27 of this lesson.

   (a) $3\frac{1}{2} \times 4 = \frac{7}{2} \times \frac{4}{1} = \frac{7 \times \cancel{4}^{2}}{\cancel{2}_{1} \times 1} = \frac{7 \times 2}{1 \times 1} = \frac{14}{1} = 14$

   (b) $4\frac{3}{4} \times \frac{1}{2} =$ = = =

   (c) $3\frac{1}{2} \times \frac{4}{7} =$ = = =

   (d) $\frac{2}{3} \times 2\frac{1}{2} =$ = = =

   (e) $2\frac{5}{16} \times 4 =$ = = =

   (f) $9\frac{1}{3} \times 12 =$ = = =

   (g) $1\frac{1}{2} \times 2\frac{3}{4} =$ = = =

   (h) $5\frac{3}{8} \times 4\frac{4}{5} =$ = = =

   (i) $4 \times \frac{3}{4} \times 7\frac{1}{2} =$ = = =

2. The cooling system of a truck has a capacity of $5\frac{1}{4}$ gallons. If the cooling system contains a solution which is $\frac{7}{16}$ antifreeze, how many gallons of antifreeze are in the system?

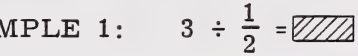
3. A rod is $16\frac{1}{2}$ feet in length. How many feet are in $3\frac{1}{2}$ rods?

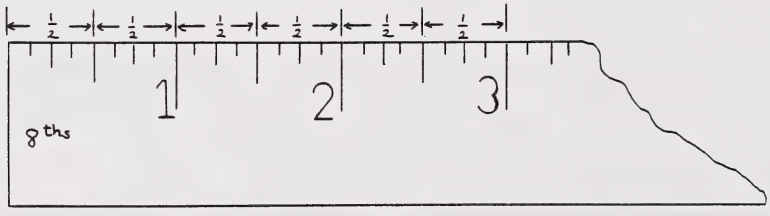
4. The cruising speed of a jet plane is 620 miles per hour. At that rate, how far will it travel in $4\frac{3}{4}$ hours?

Topic Two: Division of Fractions

A. Division of Common Fractions

Division of a number by a fraction can be illustrated by using measures.

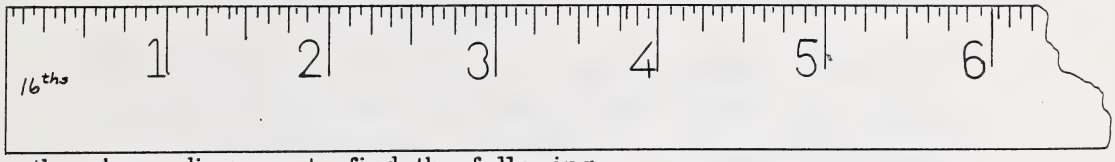
EXAMPLE 1: $3 \div \frac{1}{2} =$ 



How many $\frac{1}{2}$'s are there in 3"? From the diagram we see there are six $\frac{1}{2}$'s in 3".

$$3 \div \frac{1}{2} = 6 \qquad (3 \times 2 = 6)$$

EXERCISE:



Use the above diagram to find the following:

1. $6 \div \frac{3}{8} =$

2. $5 \div \frac{1}{4} =$

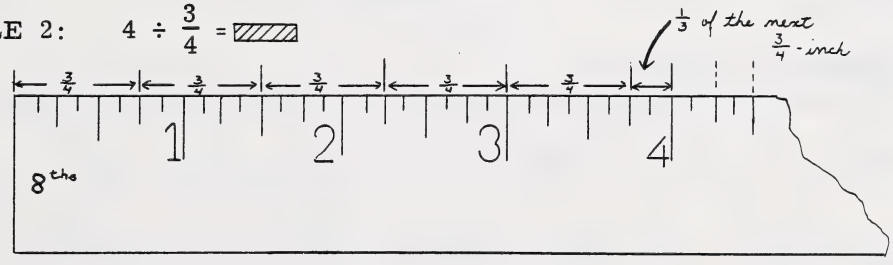
3. $3 \div \frac{3}{16} =$

4. $4 \div \frac{1}{8} =$

5. $6 \div \frac{3}{16} =$

6. $2 \div \frac{1}{16} =$

EXAMPLE 2: $4 \div \frac{3}{4} =$ ▨



There are $5\frac{1}{3}$ groups of $\frac{3}{4}$ inches in 4".

We should be able to find a quotient without the use of a diagram. Considering the above example, proeceed as follows:

1. Find how many $\frac{3}{4}$'s in 1. From our knowledge of multiplicative inverses, we know the answer is $\frac{4}{3}$.

2. Find how many $\frac{3}{4}$'s in 4. We would expect 4 times as many as in 1. Thus, multiply $\frac{4}{3}$ by 4.

3. Therefore, $4 \div \frac{3}{4} = 4 \times \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$

The examples illustrate a rule for dividing fractions.

> To divide a whole number by a fraction or a fraction by a fraction, "invert" the divisor (i.e. give the reciprocal of the divisor) and proceed as for mulitiplication of common fractions.
>
> $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$
>
> divisor

EXAMPLE 1: $\frac{25}{1} \div \frac{5}{1} = \frac{25^{5}}{1} \times \frac{1}{5_{1}} = \frac{5}{1} = 5$

This is the reciprocal of $\frac{5}{1}$.

(Compare this with 25 ÷ 5)

EXAMPLE 2: $\frac{4}{7} \div \frac{2}{9} = \frac{4}{7} \times \frac{9}{2} = \frac{4^{2}}{7} \times \frac{9}{2_{1}} = \frac{18}{7} = 2\frac{4}{7}$

divisor — reciprocal of divisor

EXERCISES:

1. Simplify the following:
   The answers to this exercise can be found on page 28 of this lesson.

   (a) $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$

   (b) $16 \div \frac{1}{4} =$

   (c) $\frac{2}{3} \div 3 =$

   (d) $\frac{5}{8} \div \frac{3}{4} =$

   (e) $\frac{1}{2} \div \frac{1}{2} =$

   (f) $\frac{1}{2} \div \frac{1}{4} =$

   (g) $3 \div \frac{1}{2} =$

   (h) $\frac{1}{8} \div 2 =$

2. Using the fact that a fraction is a quotient, simplify the following:

(a) $\dfrac{\frac{3}{4}}{\frac{1}{2}} = \dfrac{3}{4} \div \dfrac{1}{2} =$ (Fraction bar means "divide")

(b) $\dfrac{8}{\frac{1}{2}} =$

(c) $\dfrac{\frac{5}{16}}{\frac{3}{8}} =$

(d) $\dfrac{1}{\frac{1}{8}} =$

(e) $\dfrac{\frac{3}{4}}{\frac{9}{10}} =$

3. A pile of boards $\frac{3}{4}$ inch thick is 8 feet high. How many boards are in the pile?
(You must first find how many inches are in 8 feet.)

4. If a drill advances into the work $\frac{1}{64}$" each turn, how many turns are necessary to advance through 3" of material?

B. Division of Mixed Numbers

Division of mixed numbers is very simple once you know how to convert mixed numbers to their equivalent improper fractions, and also, how to divide fractions.

EXAMPLE:

$$7\frac{1}{2} \div 3\frac{3}{4} = \frac{15}{2} \div \frac{15}{4} = \frac{15}{2} \times \frac{4}{15} = \frac{\cancel{15}^{1} \times \cancel{4}^{2}}{\cancel{2}_{1} \times \cancel{15}_{1}} = \frac{1 \times 2}{1 \times 1} = \frac{2}{1} = 2$$

Change mixed numbers to improper fractions

Multiply as before

To divide mixed numbers:

(i) Change the mixed numbers to their equivalent improper fractions.

(ii) Proceed as for division of common fractions.

EXERCISES:

1. Simplify the following. All your work should be shown following the pattern of the first question.

(a) $7\frac{1}{3} \div 2\frac{1}{2} = \frac{22}{3} \div \frac{5}{2} = \frac{22}{3} \times \frac{2}{5} = \frac{22 \times 2}{3 \times 5} = \frac{44}{15} = 2\frac{14}{15}$

(b) $1\frac{2}{3} \div \frac{1}{4} = \frac{5}{3} \div \frac{1}{4} = \frac{5}{3} \times \frac{4}{1} =$

(c) $7\frac{3}{4} \div 2\frac{1}{2} =$

(d) $18 \div 2\frac{2}{5} =$

(e) $25\frac{1}{2} \div 1\frac{1}{3} =$

(f) $2\frac{9}{16} \div \frac{3}{8} =$

(g) $8\frac{3}{5} \div 6\frac{1}{7} =$

(h) $1\frac{1}{8} \div 6\frac{2}{5} =$

(i) $3\frac{1}{3} \div 2\frac{1}{2} =$

(j) $42\frac{1}{3} \div 16\frac{1}{3} =$

(k) $3\frac{1}{4} \div \frac{13}{7} =$

2. Perform the indicated operations in the order in which they appear. (First write each mixed number as an improper fraction. Then change each division sign to a multiplication sign and invert the number immediately after it.)

(a) $\frac{3}{4} \times 5\frac{1}{3} \div 1\frac{1}{3}$

$= \frac{3}{4} \times \frac{16}{3} \div \frac{4}{3}$ ← improper fractions

$= \frac{3}{4} \times \frac{16}{3} \times \frac{3}{4}$ ← Multiply by the reciprocal of $\frac{4}{3}$

$= \frac{\cancel{3}^{1} \times \cancel{16}^{4} \times 3}{\cancel{4}_{1} \times \cancel{3}_{1} \times \cancel{4}_{1}}$ ← product of numerators / product of denominators

$= \frac{1 \times 1 \times 3}{1 \times 1 \times 1}$

$= 3$

(b) $15 \div 1\frac{1}{4} \times \frac{7}{2}$

$= 15 \div \frac{\quad}{4} \times \frac{7}{2}$

$= 15 \times \frac{\quad}{5} \times \frac{7}{2}$

$=$

(c) $\frac{5}{8} \div 10 \times \frac{2}{3}$

$=$

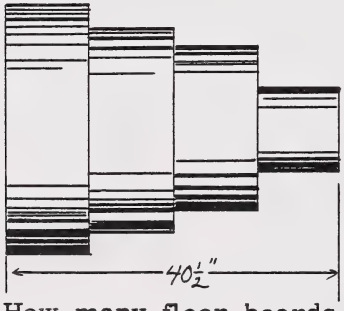
(d) $1\frac{1}{48} \times 3\frac{1}{21} \div 4\frac{2}{3}$

$=$

3. How many $1\frac{3}{4}$" machine bolt blanks can be cut from a 35" length of stock?

4. A bar of metal $3\frac{1}{2}$" long is divided into four equal parts. How long is each part?

5. The cone-step pulley (as shown in the diagram) has four equal steps. What is the width of each step?



6. How many floor boards, each $2\frac{1}{4}$" wide, will be needed to fit a floor space of $49\frac{1}{2}$"?

Topic Three: Complex Fractions (OPTIONAL)

A. What is a Complex Fraction?

From lesson 2, we learned that a common fraction is a name of the form $\frac{a}{b}$ where 'a' is a whole number and 'b' is any whole number except zero. There are also fractions in which the numerator and/or the denominator have terms other than whole numbers.

e.g. $\dfrac{\frac{3}{4}}{4}$, $\dfrac{2}{\frac{3}{4}}$, $\dfrac{\frac{3}{4}}{\frac{4}{5}}$, $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}}$

> A fraction which has one or more fractions in its numerator, or its denominator, or in both, is called a COMPLEX FRACTION.

Give six examples of complex fractions:

(a) (c) (e)

(b) (d) (f)

B. Simplifying Complex Fractions

To simplify complex fractions, we use the fact that a fraction is an expression of division. All complex fractions can be simplified to the form of a common fraction.

EXAMPLE: $\dfrac{\frac{3}{4}}{\frac{4}{5}} = \frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{3 \times 5}{4 \times 4} = \frac{15}{16}$

Fraction bar means "divide"

common fraction

(OPTIONAL)

EXERCISES: Using the fact that a fraction is a quotient, simplify the following. Your solutions should follow the pattern of the example.

(a) $\dfrac{\frac{9}{8}}{3} = \frac{9}{8} \div 3 = \frac{9}{8} \times \frac{1}{3} =$

(b) $\dfrac{\frac{1}{4}}{\frac{2}{3}} = \frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \text{—} =$

(c) $\dfrac{4}{\frac{1}{5}} =$

(d) $\dfrac{\frac{5}{9}}{\frac{9}{5}} =$

(e) $\dfrac{\frac{1}{2}}{\frac{9}{10}} =$

(f) $\dfrac{1\frac{1}{2}}{3} = 1\frac{1}{2} \div 3 =$

(g) $\dfrac{2\frac{1}{4}}{1\frac{3}{8}} = 2\frac{1}{4} \div 1\frac{3}{8} = \frac{9}{4} \div \text{——} = \frac{9}{4} \times \text{——} =$

(h) $\dfrac{1\frac{1}{9}}{2\frac{1}{2}} =$

(i) $\dfrac{4\frac{1}{4}}{5\frac{4}{5}} =$

(OPTIONAL)

Some complex fractions have more than one term in the numerator and/or denominator. In simplifying such fractions, we first perform any operations that are indicated above or below the long horizontal line. (i.e. we write the numerator and denominator as single fractions.) In the last step, we perform the operation of division which is indicated by the long horizontal line.

EXAMPLE 1:

$$\frac{\frac{1}{3} + \frac{1}{4}}{\frac{1}{3}}$$

Perform this addition first.

$$= \frac{\frac{4}{12} + \frac{3}{12}}{\frac{1}{3}}$$

$\left(\frac{4}{12} + \frac{3}{12} = \frac{4+3}{12}\right)$

$$= \frac{\frac{7}{12}}{\frac{1}{3}}$$

(The numerator and the denominator have been simplified as much as possible.)

$$= \frac{7}{12} \div \frac{1}{3}$$

Long horizontal line means "divide".

$$= \frac{7}{12} \times \frac{3}{1}$$

$$= \frac{7 \times 3^{1}}{\cancel{12}_{4} \times 1}$$

$$= \frac{7}{4}$$

$$= 1\frac{3}{4}$$

EXAMPLE 2:

$$\frac{\frac{1}{2} + \frac{3}{4}}{1 - \left(\frac{1}{2} \times \frac{3}{4}\right)} = \frac{\frac{2+3}{4}}{1 - \frac{3}{8}} \quad \left(\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4}\right)$$

$$= \frac{\frac{5}{4}}{\frac{5}{8}} \quad \text{(The division can now be performed.)}$$

$$= \frac{5}{4} \div \frac{5}{8}$$

$$= \frac{5}{4} \times \frac{8}{5}$$

$$= 2$$

EXERCISE: Simplify the following expressions. (Remember: the long horizontal line means "divide".)

(a) $\dfrac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{16}}$

$= \dfrac{\frac{1}{8}}{\frac{1}{16}}$

$= \frac{1}{8} \div \frac{1}{16}$

$=$

(b) $\dfrac{\frac{2}{5} + \frac{1}{4}}{\frac{1}{2} + \frac{4}{5}}$

$= \dfrac{\frac{\quad}{20} + \frac{\quad}{20}}{\frac{\quad}{10} + \frac{\quad}{10}}$

$= \dfrac{\frac{\quad}{20}}{\frac{\quad}{10}}$

$= \frac{\quad}{20} \div \frac{\quad}{10}$

$= \frac{\quad}{20} \times \frac{\quad}{\quad}$

$=$

(OPTIONAL)

(c) $\dfrac{\frac{1}{8} + \frac{3}{5}}{\frac{3}{4} - \frac{2}{5}}$

$= \dfrac{\frac{\quad}{40} + \frac{\quad}{40}}{\frac{\quad}{20} - \frac{\quad}{20}}$

=

(d) $\dfrac{\frac{5}{6}}{\frac{1}{2} - \frac{1}{3}}$

=

(e) $\dfrac{3 + \left(\frac{1}{2} \times 4\right)}{2 - 1\frac{1}{2}}$

$= \dfrac{3 + \quad}{\frac{\quad}{2}}$

=

(f) $\dfrac{\left(3\frac{1}{6} + 2\frac{5}{6}\right) \div \frac{2}{3}}{\frac{1}{2} \times 3\frac{1}{4} \div 6\frac{1}{2}}$

=

(OPTIONAL)

(g) $\dfrac{4 \times 3\frac{1}{7} \times \frac{2}{3}}{3 \times \frac{2}{3} \div 1\frac{1}{3}}$

=

(h) $1 + \dfrac{1}{\frac{2}{3} + \frac{2}{5}}$

=

(i) $\dfrac{8 \times \frac{9}{8}}{1 - \left(\frac{2}{3} - \frac{1}{4}\right)}$

=

(j) $\dfrac{6 + 7\frac{1}{2}}{5 \div \frac{5}{8}}$

=

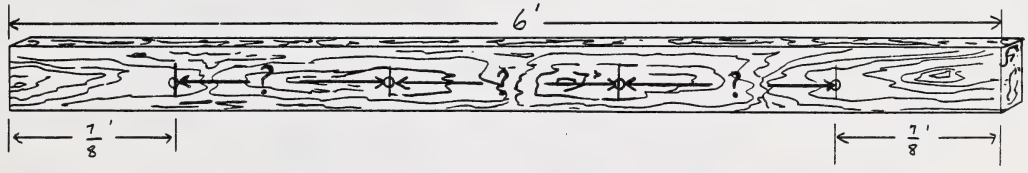
C. Problems Involving Complex Fractions (OPTIONAL)

Practical problems often involve not only addition and subtraction, but also division. The solving of such problems involves complex fractions.

EXAMPLE: If 4 holes are to be drilled in a 6-foot long piece of $2'' \times 4''$, the outer holes being $\frac{7}{8}$ feet from each end, what will be the spacing of each hole?
(Set up a complex fraction to show this relationship.)

> In solving any problem, a diagram of the situation usually helps in understanding the problem.

DIAGRAM:



Referring to the diagram, we see:

1. The total length on which the holes are to be drilled is $\left[6' - 2\left(\frac{7'}{8}\right)\right]$.
2. There will be 3 equal spaces between the holes.

   i.e. $\left[6' - 2\left(\frac{7'}{8}\right)\right]$ will be divided into 3 equal parts.

Solution

Let x equal the spacing of each hole.
The total length on which the holes are to be drilled is $6 - 2\left(\frac{7}{8}\right)$.

$$x = \frac{6 - 2\left(\frac{7}{8}\right)}{3} = \frac{6 - \frac{7}{4}}{3}$$

$$= \frac{17}{4} \div 3$$

$$= \frac{17}{4} \times \frac{1}{3}$$

$$= \frac{17}{12}$$

$$= 1\frac{5}{12}$$

There will be $1\frac{5}{12}$ feet between each hole.

(OPTIONAL)

PROBLEM 1: A steel bar is known to weigh $1\frac{7}{8}$ lb. per foot. What is the length of the piece weighing $9\frac{1}{2}$ lb.?
(You want to find how many equal lengths of $1\frac{7}{8}$ pounds each are in the bar.)

Solution:

Let 'x' equal the length of the bar

x =

The length of the bar is _____ feet.

PROBLEM 2: A box contains bolts each of which weighs $\frac{1}{8}$ pound. The box of bolts weighs $48\frac{1}{2}$ pounds and the weight of the box when empty is $4\frac{1}{4}$ pounds. How many bolts are in the box?

Solution:

Let 'n' equal the number of bolts.
The total weight of the bolts would be _____ pounds.

n =

There are _____ bolts in the box.

Answers to Self-Correcting Exercises on Page 4 of this Lesson

1. (a) $\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7} = \boxed{\frac{6}{35}}$

(b) $\frac{2}{5} \times \frac{1}{3} = \frac{2 \times 1}{5 \times 3} = \boxed{\frac{2}{15}}$

(c) $\frac{15}{16} \times \frac{3}{4} = \frac{15 \times 3}{16 \times 4} = \boxed{\frac{45}{64}}$

(d) $\frac{3}{5} \times 3 = \frac{3 \times 3}{5 \times 1} = \frac{9}{5} = \boxed{1\frac{4}{5}}$

($\frac{9}{5}$ is an improper fraction)

(e) $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \boxed{\frac{8}{15}}$

(f) $\frac{1}{4} \times \frac{7}{8} = \frac{1 \times 7}{4 \times 8} = \boxed{\frac{7}{32}}$

(g) $\frac{1}{8} \times 9 = \frac{1 \times 9}{8 \times 1} = \frac{9}{8} = \boxed{1\frac{1}{8}}$

($\frac{9}{8}$ is simplified to $1\frac{1}{8}$)

(h) $\frac{5}{6} \times \frac{1}{4} = \frac{5 \times 1}{6 \times 4} = \boxed{\frac{5}{24}}$

(i) $\frac{3}{10} \times \frac{3}{2} = \frac{3 \times 3}{10 \times 2} = \boxed{\frac{9}{20}}$

(j) $\frac{10}{3} \times \frac{2}{3} = \frac{10 \times 2}{3 \times 3} = \frac{20}{9} = \boxed{2\frac{2}{9}}$

($\frac{20}{9} = \frac{18}{9} + \frac{2}{9} = 2 + \frac{2}{9}$)

Answers to Self-Correcting Exercises on Page 11 of this Lesson

1. (a) $3\frac{1}{2} \times 4 = \frac{7}{2} \times \frac{4}{1} = \frac{7 \times \cancel{4}^{2}}{\cancel{2}_{1} \times 1} = \frac{7 \times 2}{1 \times 1} = \frac{14}{1} = \boxed{14}$

(b) $4\frac{3}{4} \times \frac{1}{2} = \frac{19}{4} \times \frac{1}{2} = \underbrace{\frac{19 \times 1}{4 \times 2}} = \frac{19}{8} = \boxed{2\frac{3}{8}}$

no common factors

(c) $3\frac{1}{2} \times \frac{4}{7} = \frac{7}{2} \times \frac{4}{7} = \underbrace{\frac{\cancel{7} \times \cancel{4}^{2}}{\cancel{2}_{1} \times \cancel{7}_{1}}} = \frac{1 \times 2}{1 \times 1} = \frac{2}{1} = \boxed{2}$

(common factors 2 and 7 are cancelled)

(d) $\frac{2}{3} \times 2\frac{1}{2} = \frac{2}{3} \times \frac{5}{2} = \frac{\cancel{2}_{1} \times 5}{3 \times \cancel{2}_{1}} = \frac{1 \times 5}{3 \times 1} = \frac{5}{3} = \boxed{1\frac{2}{3}}$

Rename improper fraction as a mixed number

(e) $2\frac{5}{16} \times 4 = \frac{37}{16} \times \frac{4}{1} = \frac{37 \times \cancel{4}^{1}}{\cancel{16}_{4} \times 1} = \frac{37 \times 1}{4 \times 1} = \frac{37}{4} = \boxed{9\frac{1}{4}}$

($37 \div 4 = 9$, remainder 1)

(f) $9\frac{1}{3} \times 12 = \frac{28}{3} \times \frac{12}{1} = \frac{28 \times \cancel{12}^{4}}{\cancel{3}_{1} \times 1} = \frac{28 \times 4}{1 \times 1} = \frac{112}{1} = \boxed{112}$

(Common factor 3 cancels out)

(g) $1\frac{1}{2} \times 2\frac{3}{4} = \frac{3}{2} \times \frac{11}{4} = \underbrace{\frac{3 \times 11}{2 \times 4}} = \frac{33}{8} = \boxed{4\frac{1}{8}}$

no common factors

(h) $5\frac{3}{8} \times 4\frac{4}{5} = \frac{43}{8} \times \frac{24}{5} = \frac{43 \times \cancel{24}^{3}}{\cancel{8}_{1} \times 5} = \frac{43 \times 3}{1 \times 5} = \frac{129}{5} = \boxed{25\frac{4}{5}}$

(Common factor 8 cancels out)

(i) $4 \times \frac{3}{4} \times 7\frac{1}{2} = \frac{4}{1} \times \frac{3}{4} \times \frac{15}{2} = \frac{\cancel{4}^{1} \times 3 \times 15}{1 \times \cancel{4}_{1} \times 2} = \frac{1 \times 3 \times 15}{1 \times 1 \times 2} = \frac{45}{2} = \boxed{22\frac{1}{2}}$

Answers to Self-Correcting Exercise on Page 14 of this Lesson

(a) $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \boxed{\frac{1}{4}}$

(b) $16 \div \frac{1}{4} = \frac{16}{1} \times \frac{4}{1} = \frac{16 \times 4}{1 \times 1} = \boxed{64}$

$\left(16 = \frac{16}{1}\right)$

(c) $\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3} = \frac{2 \times 1}{3 \times 3} = \boxed{\frac{2}{9}}$

(3 inverted is $\frac{1}{3}$)

(d) $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5 \times \cancel{4}^{1}}{\cancel{8}_{2} \times 3} = \frac{5 \times 1}{2 \times 3} = \boxed{\frac{5}{6}}$

(e) $\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times \frac{2}{1} = \frac{1 \times \cancel{2}^{1}}{\cancel{2}_{1} \times 1} = \boxed{1}$

(Common factor of 2 cancels)

(f) $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{1 \times \cancel{4}^{2}}{\cancel{2}_{1} \times 1} = \frac{1 \times 2}{1 \times 1} = \frac{2}{1} = \boxed{2}$

(g) $3 \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = \frac{3 \times 2}{1 \times 1} = \frac{6}{1} = \boxed{6}$

(h) $\frac{1}{8} \div 2 = \frac{1}{8} \times \frac{1}{2} = \frac{1 \times 1}{8 \times 2} = \boxed{\frac{1}{16}}$

End of Lesson 4

Practical Mathematics
Module 2
Lesson 5

PROBLEM SOLVING

Topic One: Approaching a Problem

In any trade, the ability to solve problems is very important.

Tradesmen are all concerned with quantities involved in measurement of distance, area, volume, weight, time, rotation, and something that interests us all, money value. These are the things problem solving is concerned with.

Difficulties which arise in problem solving stem from:
1. Understanding the problem — i.e. what are you asked to find and what information is essential to the solution?
2. choosing the operation(s) to be used
3. selecting and using the appropriate units of measure.

Topic Two: Choosing the Operation

I. Four Fundamental Operations

There are four fundamental operations involved with problem solving: addition, subtraction, multiplication, division. Recognizing expressions which involve the operation is a big step towards the solution of a problem.

Study the following terms which refer to the four fundamental operations.

+ (Addition)	- (Subtraction)
plus	minus
sum of	difference between
added to	subtracted from
increased by	diminished by
together we have	less than
more than	

× (Multiplication)	÷ (Division)
times	divided by
product of	quotient of
multiplied by	fractional part of
of	one number over another

EXERCISE 1: Rewrite the following phrases using symbols to indicate the operation being expressed.

EXAMPLE: 15 increased by 6 $15 + 6$

(a) 17 more than 20 _____

(b) the difference between 5 and 2 _____

(c) the product of 10 and 11 _____

(d) one half of 18 _____

(e) 17 over 6 _____

(f) 15 less than 20 _____

(g) 19 increased by 1 _____

(h) $\frac{1}{4}$ divided by $\frac{1}{2}$ _____

(i) $\frac{7}{8}$ more than 5 _____

(j) $\frac{1}{2}$ is diminished by $\frac{3}{8}$ _____

(k) the quotient of 2 and $\frac{1}{3}$ _____

EXERCISE 2: Rewrite the following sentences using a letter to represent the unknown quantity, and mathematical symbols to represent the operation involved.

EXAMPLE: An unknown quantity is equal to 7 more than 5.

$x = 5 + 7$

(a) $\frac{7}{8}$ added to $\frac{1}{2}$ is equal to an unknown quantity.

_____

(b) 8 increased by $\frac{2}{3}$ is $\frac{3}{4}$ of some unknown number.

_____

(c) The quotient of 9 and $\frac{1}{9}$ is equal to some unknown.

_____

(d) Some unknown is equal to 17 diminished by $8\frac{1}{2}$.

_____

(e) One half of an unknown quantity is equal to $2\frac{1}{2}$.

_____

(f) Taking 5 and 15, together we have an unknown quantity.

_____

(g) 19 less than 100 is equal to some unknown.

_____

(h) $\frac{5}{8}$ of 50 feet equals an unknown quantity.

_____

(i) $\frac{1}{4}$ subtracted from some number is equal to an unknown quantity.

_____

(j) 10 times 15 is equal to some unknown quantity.

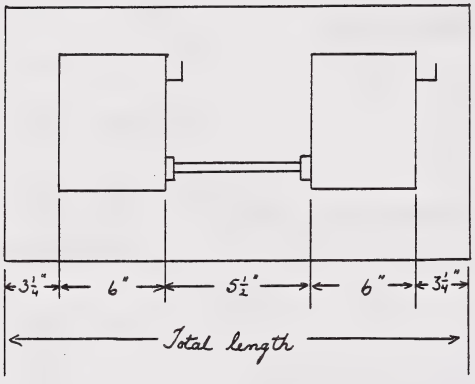
_____

II. Specific Cases Involving the Fundamental Operations

(a) Addition: Addition is necessary when a total length is needed.

EXAMPLE:

If an electrician wants to know how long a piece of wood is needed to support a meter panel, he must add the length of each part of the panel to obtain the total length.



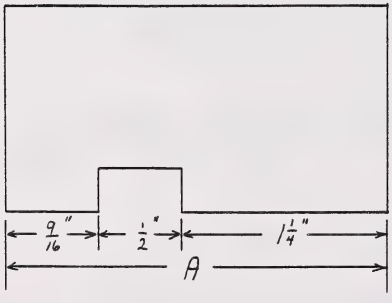
The following additions must be performed:

Total length = $3\frac{1}{4}'' + 6'' + 5\frac{1}{2} + 6'' + 3\frac{1}{4}''$

= 24"

The length of the piece of wood needed is 24 inches.

PROBLEM: Find the overall length A in the following blueprint sketch. (Show the operation to be done.)



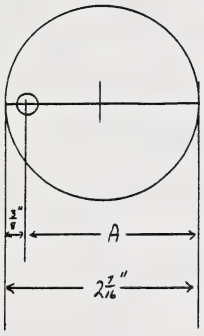
A =

=

The overall length is _____ inches.

(b) Subtraction: The worker in industry is often confronted with the problem of finding a length by subtraction.

EXAMPLE: The diagram shows an eccentric (off center) cam. Find the distance A.



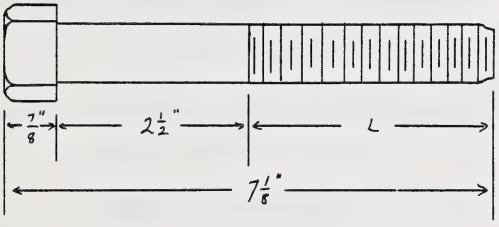
The distance A is less than the total distance, therefore, the operation used is subtraction.

$A = 2\frac{7}{16}" - \frac{3}{8}"$

$=$

The distance A is _____ inches.

PROBLEM: Find the distance, L, for the bolt.



You must first find the length $\left(\frac{7}{8}" + 2\frac{1}{2}"\right)$.

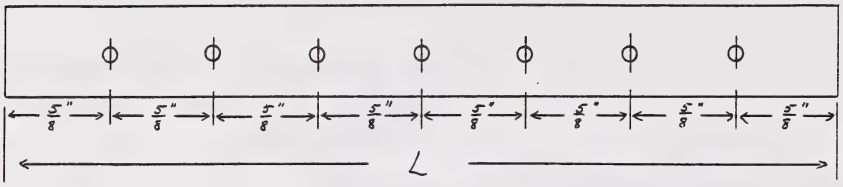
$L =$ _____ $- \left(\frac{7}{8}" + 2\frac{1}{2}"\right)$

$=$

The unknown length is _____ inches.

(c) Multiplication: Multiplication is a short cut for adding quantities of the same value.

e.g. A piece of sheet metal is punched with holes. The holes form eight spaces each $\frac{5}{8}$" apart (i.e. there are eight spaces of $\frac{5}{8}$"). To find L, multiply $8 \times \frac{5}{8}$.



$$L = 8 \times \frac{5}{8}"$$

$$= 5"$$

The overall length is 5 inches.

PROBLEM: A carpenter needs 15 lengths of $\frac{3}{4}$" lumber. Each length is 12 feet. What is the total length needed?

(The $\frac{3}{4}$" is not needed for the solution. The length of lumber remains the same regardless of the size of lumber.)

* The final answer to any problem should be given in a sentence.

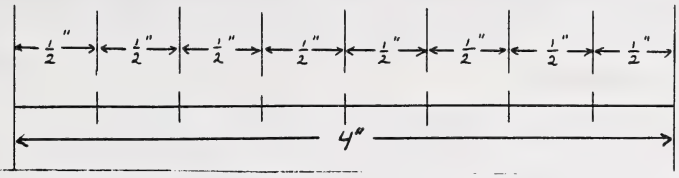
Total length =

= _____ feet

Final statement: _____

(d) Division: Division is used when dividing a whole into equal parts.

e.g. 4" is divided into $\frac{1}{2}$"'s. How many equal parts are there?



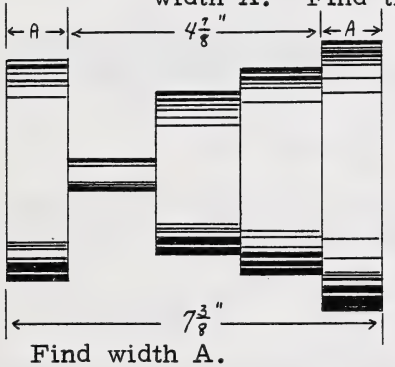
4 inches is divided by $\frac{1}{2}$ inch.

Number of parts = $4" \div \frac{1}{2}"$

= 8

There are _____ equal parts.

PROBLEM: A pulley has 3 equal steps. The end distances are of equal width A. Find the width of each step and width A.



Find width A.

Width of each step =

The width of each step is _____ inches.

(Taking the two A's together, find how much less these widths are than the total length.)

Width of both A's =

=

Width of A =

=

The width of A is _____ inches.

Topic Three: Units of Measure

1. Knowing the units of measure (or the name of the answer) is essential to the complete solution of any problem. Whenever possible, it is useful to write the unit of measurement after each number as you are doing the operations.

   To determine the final unit of measurement, consider the following:

   (a) Addition — When adding, the answer will be in the same units of measure as the units being added.
   e.g. 7 inches + 3 inches = 10 inches

   (b) Subtraction — When subtracting, the answer will be in the same units of measure as the units being subtracted.
   e.g. 110 volts - 32 volts = 78 volts

   (c) Multiplication — When multiplying units of measurement by a number with no dimension, the answer will be in the same unit.
   e.g. \$7.20 × 38 = \$273.60

   (d) Division — (a) When dividing units of measure by the same unit, the answer will be a number only.
   e.g. 8 inches ÷ $\frac{1}{2}$ inch = 16

   (b) When dividing units of measure by a number with no dimension, the answer will be in the same unit of measure.
   e.g. 20 feet ÷ $\frac{1}{4}$ = 80 feet

EXERCISE: Supply the correct answer (along with the correct units of measure) for the following operations.

(a) $\frac{9}{8}$ feet + $\frac{7}{8}$ feet = _____

(b) $7 \times \frac{5}{16}$ inches = _____

(c) 20 amperes - 7 amperes = _____

(d) \$4000 ÷ \$400 = _____

(e) 32 r.p.m. ÷ 4 = _____

2. Frequently the mechanic (as well as other tradesmen) finds it necessary to solve problems involving mixed units, that is, inches, feet and yards. Numbers that represent mixed units may be added, subtracted, or multiplied, but you must be careful to keep the units separate.

(a) Addition:

$$\begin{array}{rrr} & 6' & 7'' \\ + & 14' & 5'' \\ \hline & 20' & 12'' \end{array}$$

← inches are added.

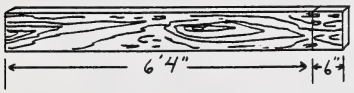
Feet are added

If necessary, the final answer can be converted to simpler units.
i.e. 20' 12" = 21' since 12" = 1'

*Conversions are dealt with in a later module and need not be worried about at this point.

PROBLEM: If you joined a 6 foot 4 inch length of lumber to a 6 inch piece of lumber, what would the overall length be?

Overall length =



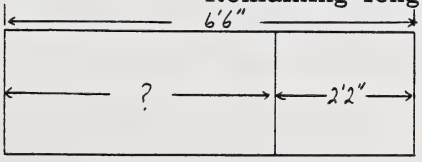
The length would be _____.

(b) Subtraction:

$$\begin{array}{rrr} & 10' & 6'' \\ - & 4' & \\ \hline & 6' & 6'' \end{array}$$

PROBLEM: A 2 foot 2 inch piece of sheet metal is cut off a piece 6 feet 6 inches long. How long is the remaining piece?

Remaining length =



=

The remaining piece is _____ long.

(c) Multiplication:

$$\begin{array}{rr} 3 \text{ yards} & 2 \text{ feet} \\ \times & 3 \\ \hline 9 \text{ yards} & 6 \text{ feet} \end{array}$$

PROBLEM: Three boards each 1 foot 3 inches long are nailed together end-to-end. What is the length of the boards once they are nailed together? (Draw yourself a diagram.)

Length =

Final statement: _____

(d) Division: Division of mixed units presents a different case which involves conversions and will not be dealt with at this time. Only an example is given.

EXAMPLE: (5 feet 6 inches) ÷ 2 = (60 inches + 6 inches) ÷ 2
= (66 inches) ÷ 2
= 33 inches

(or) = 2 feet 9 inches

Topic Four: Setting Up a Problem

Your success in problem solving will increase as you learn to recognize and use a systematic approach such as the following:

1. Read the problem carefully.
   Before approaching a problem, you must clearly understand what the problem is asking you to find. You must be clear what information is necessary for the complete solution of the problem.

2. Draw a diagram and include dimensions whenever possible.
   This gives a clearer picture of what the problem is all about.

3. Write clear statements to indicate:
   (a) the quantity,
   (b) the unit of measurement,
   (c) the required measure expressed in terms of the known measure.

4. Simplify the measures and find the answer in an orderly manner.

5. Include the final answer in a sentence which tells what the original problem was asking for.

6. Check to see if your answer is reasonable.
   Make an estimate of your final answer by rounding the figures involved in the problem.

Now study the following examples and note how the problem-solving approach has been used. Fill in any blanks that occur in the examples.

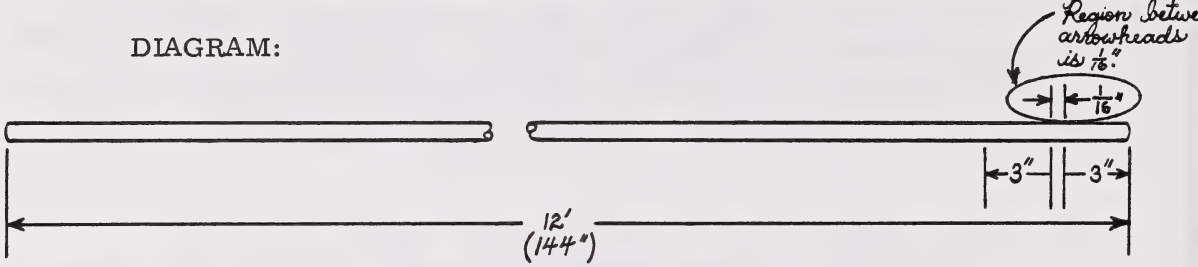
EXAMPLE 1:

For a certain job, steel pins 3 inches long are required.

(a) How many of these pins can be cut from a piece of stock 12 feet long if $\frac{1}{16}$" is wasted in each cut? (12' = 144")

(b) What length of stock is left after the pins have been cut off?

DIAGRAM:



(a) (From the problem, we need to know how much stock is cut off for each pin.)

The length of stock required for each pin is 3 in. + $\frac{1}{16}$ in.

(Note how the quantities and units of measure are indicated.)

= _____ in.

("New" measure is expressed in terms of the known measure.)

The no. of pieces cut off is 144 ÷ _____ (Simplifying the measure)

$144 \div \frac{49}{16}$

$= \frac{144}{1} \times \frac{16}{49}$

144
16
864
144
2304

47
49)2304
196
344
343
1 R

= $47\frac{1}{49}$ pieces

CHECK: The no. of pins = 150 ÷ 3 = _____

We rounded 144 to 150

Note that $47\frac{1}{49}$ is close to this figure

(b) The length of the remaining stock is $\frac{1}{49}$ of $3\frac{1}{16}$ in.

= _____.

($\frac{1}{49}$ gives you the part of a pin which remains)

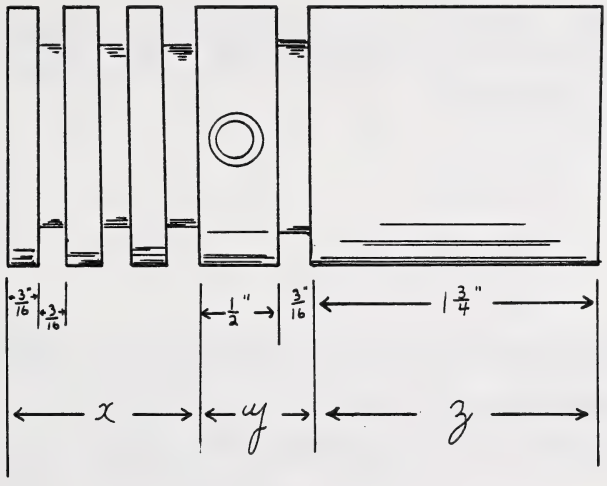
CHECK: Length = $\frac{1}{48} \times 3''$ = _____.

($\frac{1}{48}$ and 3 are close to the figures involved and are easy to compute with.)

Estimated answer should be close to the real answer

EXAMPLE 2:

From the diagram of a piston
(a) calculate the measures represented by x and y.
(b) find the overall length of the piston.



(a) The measure of x = $6 \times \frac{3}{16}$" = _____.

The measure of y = $\frac{1}{2}$" + $\frac{3}{16}$" = _____.

The measure of z = _____.

The measures represented by x, y, and z are _____,

_____, and _____ respectively.

(Are your answers reasonable?)

(b) Overall length = _____ = _____

The overall length of the piston is _____.

CHECK: (Round the figures you obtained in 'a' to the nearer inch.)

Length = _____ + _____ + _____

= _____

(Compare estimated answer and real answer.)

Topic Five: Practice Solving Problems

Solve each of the following problems using an approach similar to that discussed in the last section. For each problem, be sure to give the answer in a sentence and to include the units of measure along with the answers.

1. Four shipments of lumber were received by a retailer: 8,662; 7,198; 4,146 and 9,123 board feet respectively. How many board feet of lumber were delivered?

2. If $5\frac{1}{3}$ pints of transformer oil are poured from a gallon can full of oil, how much remains?
(Note: There are 8 pints in one gallon.)

(Is your answer reasonable? _____)

3. A school has 20 rooms. Each room has 1200 square feet of surface area to be painted. The cost of painting is estimated at $45 per 1000 square feet. How much will it cost to paint 20 rooms?

Total area to be painted = 20 × _____ square feet.

= _____ square feet

No. of 1000 sq.ft. in total area = _____ ÷ _____

= _____

Cost =

FINAL STATEMENT: _____

CHECK: (Use Estimates.)

4. An iron-worker machine is to be used to cut 6-inch, 8-inch or 10-inch angle-iron braces. What is the shortest length of angle iron that would serve for cutting a quantity of any one of these braces without waste?

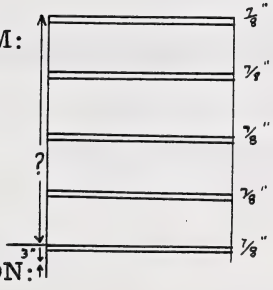
   (To solve, find the lowest common multiple of 6, 8 and 10.)

5. A book case is to be 5' 4" high (outside measurement). Five shelves each $\frac{7}{8}$ inch thick, are to be inserted at equal distances apart. Calculate the distance between the shelves if the top of the lower shelf is 3 inches from the floor.

   [5' 4" = $\underbrace{(5 \times 12)}$ in. + 4 in. = 64 in.]

   [There are 12" in a foot so there are (5×12)" in 5']

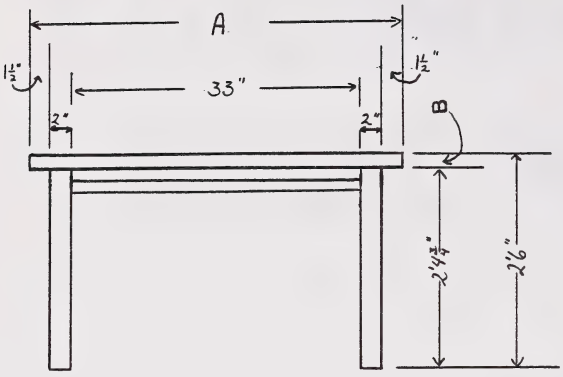
DIAGRAM:



SOLUTION:

The distance between shelves is _____.

6. Calculate dimensions A and B as indicated on the following diagram.



7. A rough iron forging weighs $52\frac{3}{4}$ pounds. When it is finished, it weighs $35\frac{1}{2}$ pounds. What is the weight of the material removed?

8. A brass rod 3' 8" is to be cut into 16 pieces. Find the length of each piece, allowing $\frac{1}{15}$" per cut for waste in cutting each piece. (Draw yourself a diagram to illustrate the problem.)

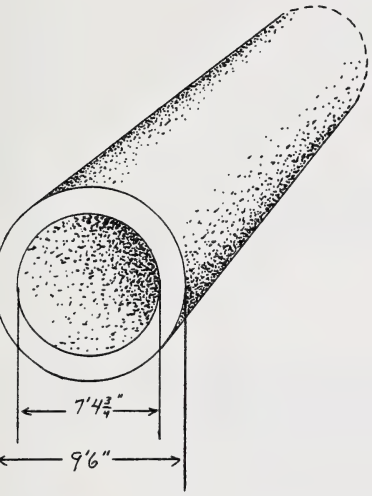
SOLUTION:

The brass rod, in inches is $(3 \times 12)" + 8"$ or _____ inches long.

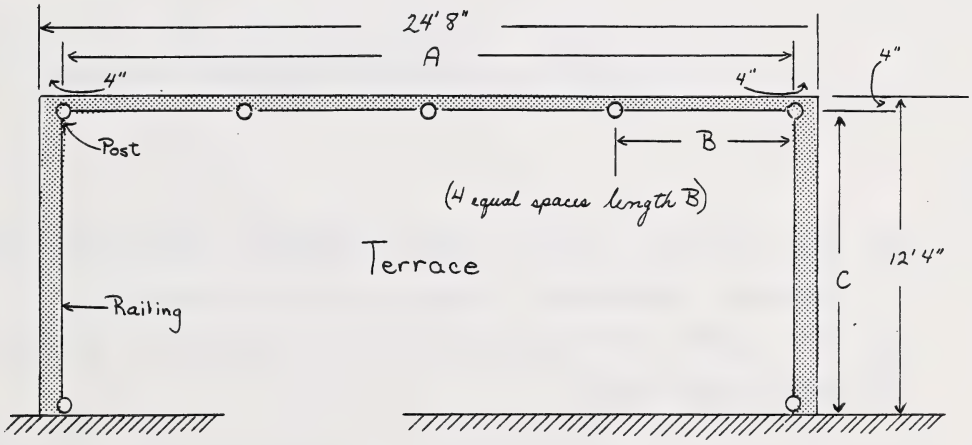
CHECK: (Use Estimates.)

9. A circular concrete tunnel has an outside diameter of 9' 6" and an inside diameter of 7' $4\frac{3}{4}$". What is the wall thickness?

(Hint: The outside diameter takes in <u>two</u> walls.)



10. In the plan of railing shown in the figure, find dimensions A, B, and C. What is the total length of railing required?

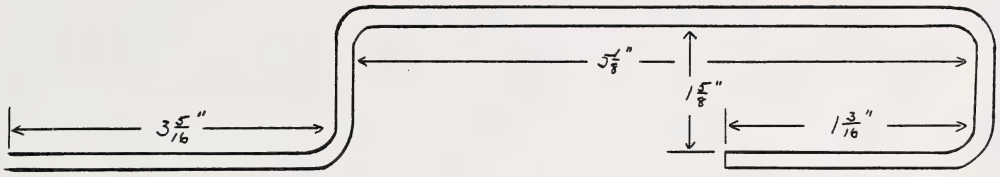


11. A gang of men mix and place an average of $38\frac{1}{2}$ cubic yards of concrete an hour. How many cubic yards do they place in a day of $7\frac{3}{4}$ hours?

12. If a drill advances into the work $\frac{1}{64}$" each turn, how many turns are necessary to advance through 3" of material?

13. The total resistance of three lights in series is obtained by adding their separate resistances. What is the total resistance of a circuit with three lights which have resistances of $3\frac{1}{2}$ ohms, $4\frac{2}{3}$ ohms and $7\frac{3}{10}$ ohms respectively?

14. Find the length of $\frac{1}{8}$" thick strap iron required to bend into the handle shown. The allowance for bending for each turn is $\frac{1}{2}$ of the thickness of the metal.



Bending allowance required for each turn = $\frac{1}{2} \times$ _____ in. = _____

Bending allowance required for all turns = $4 \times$ _____ = _____

Total length =

The length of strap iron required is _____.

CHECK: (Use Estimates.)

End of Lesson 5

Practical Mathematics
Module 2
Answer Keys

Page 1

Inventory Test

1. (a) 2969 (b) 2506 (c) 3734 (d) 4050

(e)
$$\begin{array}{r} 4909 \\ 556 \\ 9600 \\ 74 \\ +\ 150 \\ \hline 15289 \end{array}$$

(f)
$$\begin{array}{r} 1096 \\ 3321 \\ 920 \\ +9108 \\ \hline 14445 \end{array}$$

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2. (a) 229 (b) 316 (c) 287 (d) 3909 (e) 1195

(f)
$$\begin{array}{r} 1086 \\ -\ 597 \\ \hline 489 \end{array}$$

3. (a)
$$\begin{array}{r} 93 \\ \times\ 46 \\ \hline 558 \\ 372 \\ \hline 4278 \end{array}$$

(b)
$$\begin{array}{r} 45 \\ \times\ 91 \\ \hline 45 \\ 405 \\ \hline 4095 \end{array}$$

(c)
$$\begin{array}{r} 846 \\ \times\ 95 \\ \hline 4230 \\ 7614 \\ \hline 80370 \end{array}$$

(d)
$$\begin{array}{r} 805 \\ \times\ 64 \\ \hline 3220 \\ 4830 \\ \hline 51520 \end{array}$$

(e)
$$\begin{array}{r} 395 \\ \times\ 751 \\ \hline 295 \\ 1475 \\ 2065 \\ \hline 221545 \end{array}$$

(f)
$$\begin{array}{r} 701 \\ \times\ 700 \\ \hline 490700 \end{array}$$

(g)
$$\begin{array}{r} 527 \\ \times\ 856 \\ \hline 3162 \\ 2635 \\ 4216 \\ \hline 451112 \end{array}$$

(h)
$$\begin{array}{r} 948 \\ \times\ 600 \\ \hline 568800 \end{array}$$

(i)
$$\begin{array}{r} 856 \\ \times\ 208 \\ \hline 6848 \\ 000 \\ 1712 \\ \hline 178048 \end{array}$$

(j)
$$\begin{array}{r} 2629 \\ \times\ 9001 \\ \hline 2629 \\ 000 \\ 000 \\ 23661 \\ \hline 23663629 \end{array}$$

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4. (a)
$$\begin{array}{r} 6 \\ 574\overline{)4016} \\ 3444 \\ \hline 572 \end{array}$$

(b)
$$\begin{array}{r} 2 \\ 25\overline{)72} \\ 50 \\ \hline 22 \end{array}$$

(c)
$$\begin{array}{r} 8 \\ 19\overline{)162} \\ 152 \\ \hline 10 \end{array}$$

4. (d)
$$\begin{array}{r} 8 \\ 151\overline{)1269} \\ 1208 \\ \hline 61 \end{array}$$

(e)
$$\begin{array}{r} 3 \\ 600\overline{)2007} \\ 1800 \\ \hline 207 \end{array}$$

(f)
$$\begin{array}{r} 11 \\ 790\overline{)8916} \\ 790 \\ \hline 1016 \\ 790 \\ \hline 226 \end{array}$$

(g)
$$\begin{array}{r} 9 \\ 87\overline{)845} \\ 783 \\ \hline 62 \end{array}$$

(h)
$$\begin{array}{r} 14 \\ 204\overline{)3015} \\ 204 \\ \hline 975 \\ 816 \\ \hline 159 \end{array}$$

5. (a)
$$\begin{array}{r} 62 \\ 507\overline{)31746} \\ 3042 \\ \hline 1326 \\ 1014 \\ \hline 312 \end{array}$$

Answer is $62\frac{312}{507}$
- this would round to 63 (to the nearest whole number)

(b)
$$\begin{array}{r} 306 \\ 12\overline{)3677} \\ 36 \\ \hline 0077 \\ 72 \\ \hline 5 \end{array}$$

Answer is $306\frac{5}{12}$
- this would round to 306 (to the nearest whole number)

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6. (a) 8
(b) 100
(c) 63 000
(d) 9000
(e) 9
(f) 180 000
(g) 400
(h) 59
(i) 253
(j) 900

PART B

1. (b) seven hundred sixty thousand, one hundred thirty-five
   (c) ten million
   (d) five million, two hundred twenty thousand, six
   (e) nine hundred ninety one thousand, three hundred seven

Page 5

2. (a) 50 096 (b) 100 017 (c) 690 002 (d) 378 530
   (e) 200 000 005 (f) 520 032 040

3.

7	**7 ten million**
6	**6 million**
8	**8 hundred thousand**
4	**4 ten thousand**
3	**3 thousands**
2	**2 hundreds**
0	**0 tens**
1	**1 one**

Page 6

4. (a) 50 271 003 048 (b) 048 (c) billions (d) millions
   (e) 003 (f) fifty billion, two hundred seventy-one million, three thousand, forty eight

5. (a) 99 000
   (b) 930
   (c) 2000
   (d) 201 000
   (e) 1 600 000
   (f) 46 000
   (g) 1850
   (h) 9000

Page 7

1. 19 - minuend; 6 - subtrahend; 13 - difference; 8 - addend; 4 - addend; 12- sum; 15 - factor (or multiplicand); 3 - factor (multiplier); 45 - product; 9 - divisor; 334 - dividend; 37 - quotient; 1 - remainder

2. (i) $12 + 0 = 12$
(e) $3 \times 6 = 6 \times 3$
(a) $19 \times 0 = 0$
(f) $(2 + 7) + 6 = 2 + (7 + 6)$
(b) $14 + 2 = 2 + 14$
(g) $21 \times 1 = 21$
(d) $(9 + 2) \times 3 = 9 \times 3 + 2 \times 3$
(h) $(7 \times 8) \times 4 = 7 \times (8 \times 4)$

Page 8

3. (a) 0 (b) 0 (c) 0 (d) impossible (e) impossible (f) the same number
(g) 0 (j) 67 (m) 21
(h) 15 (k) 1 (n) 0
(i) 15 (l) impossible (o) 0

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Part D

(b) (i) 12: $\mathbf{2 \times 2 \times 3}$ 20: $\mathbf{2 \times 2} \times 5$
(ii) H.C.F. $= 2 \times 2 = 4$
(iii) Multiples of 12 are 12, 24, 36, 48, **60** ...
Multiples of 20 are 20, 40, **60**, 80, 100 ...
(iv) L.C.M. = **60**

(c) 9 and 15
(i) prime factors
9: 3×3 $15 = 3 \times 5$
(ii) H.C.F. = 3
(iii) Multiples of **9** are 9, 18, 27, 36, 45, 54, 63
Multiples of **15** are 15, 30, 45, 60, 75
(iv) L.C.M. = **45**

(d) 24 and 36
(i) prime factors
24: $2 \times 2 \times 2 \times 3$ 36: $2 \times 2 \times 3 \times 3$
(ii) H.C.F. is $2 \times 2 \times 3 = 12$
(iii) Multiples of 24 are 24, 48, 72, 96
Multiples of 36 are 36, 72, 108, 144
(iv) L.C.M. is **72**

Page 10

1.

	Avg. km per hr. by Train	Avg. km per hr. by Plane
Montreal - Toronto	93	558
Montreal - Vancouver	68	480
Toronto - Vancouver	67	563
Winnipeg - Calgary	107	462
Toronto - New York	75	897

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Remember there are many acceptable estimations. The estimations done here are only some of many acceptable answers.

2. Round 13 540 to 14 000 and round 225 to 200.
So approximate earnings per day would be \$14 000 ÷ 200 = \$70
So the plasterer's average pay per day is about \$70
Actual answer would be \$13 540 ÷ 225

$$
\begin{array}{r}
60.1 \\
225\overline{)13540.0} \\
\underline{1350} \\
40\,0 \\
\underline{22\,5}
\end{array}
$$

So actual average wage would be \$60 (to the nearest dollar)

2. (b) **Estimation: Journeyman:** Round \$6.50 to \$7.00 and 35 hrs. to 40 hrs.
So his weekly wage would be $7 \times 40 = \$280$

Apprentice: Round \$4.70 to \$5.00 and 35 hrs. to 40 hrs.
So the approximate weekly wage would be $\$5 \times 40 = \$200$.

So the approximate difference in their wage would be \$280 - \$200 = \$80

Actual Answer:

Journeyman's exact wage $\$6.50 \times 35 = \$227.50$
Apprentice exact wage $\$4.70 \times 35 = 164.50$
Difference \$ 63.00

Conclusion: The Journeyman earned \$63.00 more than the apprentice.

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2. (c) **Estimation:** Round 1276 to 1300
So approximate cost of building house is $1300 \times \$20 = \$26\,000.00$
So estimated cost including lot would be $\$26\,000 + 11\,000 = \$37\,000.00$

Actual Cost: Cost of house is $1276 \times \$20 = \$25\,520$

So cost of house building and lot would be
$$\begin{array}{r} \$25\,520 \\ +\ 11\,000 \\ \hline \$36\,520 \end{array}$$

Conclusion: The cost of the house and lot will be \$36 520

(d) **Estimation of Refund:**

(i) Estimation: Round \$2954 to \$3000, and round \$2309 to \$2300.
This approximate refund is \$3000 - 2300 = \$700

So he should receive about \$700.00.

(ii) **Actual Answer:** The exact value of the refund would be \$2954 - \$2309

or
$$\begin{array}{r} 2954 \\ -2309 \\ \hline \$\ 645 \end{array}$$

Conclusion: He should expect a \$645 rebate.

Page 1

1. A (a) $\frac{7}{9}$ (b) $\frac{15}{8}$ (c) $\frac{5}{6}$ (d) $\frac{10}{27}$ (e) $\frac{36}{9} = 4$ (or $\frac{4}{1}$)

B. (a) $\frac{5}{2}$ (c) $\frac{6}{1}$

(b) $\frac{7}{9}$ (d) $\frac{4}{7}$

Page 3

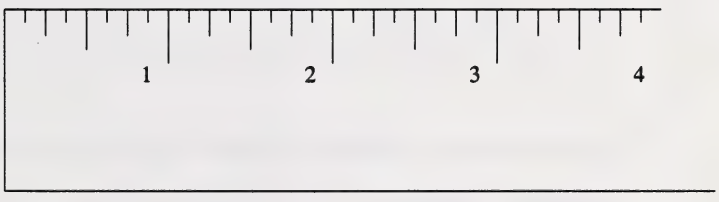
Problem 1 Solution - $\$\frac{550}{2}$ or $\$\frac{275}{1}$ (or $275.00)

Problem 2 Solution $\frac{5}{8}$ ft.

Page 4 **Exercise**

(f) $\frac{6}{4}$

(g) $\frac{0}{3}$ (a) $\frac{1}{2}$ (b) $\frac{7}{8}$ (d) $\frac{12}{8}$ (c) $\frac{7}{4}$ (e) $\frac{13}{4}$



Page 5

Complete the following table:

Fraction	Numerator	Denominator
$\frac{1}{5}$	1	5
$\frac{9}{8}$	9	8
$\frac{7}{8}$	7	8
$\frac{5}{25}$	5	25
$\frac{2}{5}$	2	5

numerator

denominator

Page 6

The first two questions will have various answers. In giving examples of **proper fractions** the numerator (top) must be smaller than the denominator (bottom) e.g. $\frac{3}{5}, \frac{2}{6}, \frac{11}{27}$ etc.

In giving examples of **improper fractions**, the numerator (top) must be greater than or equal to the denominator (bottom) e.g. $\frac{5}{4}, \frac{6}{6}, \frac{11}{4}, \frac{106}{72}$.

2. (a) denominator; numerator
   (b) numerator; denominator
   (c) greater (or more); less

3. (a) $\frac{8}{4} = 2$ (d) $\frac{4}{4} = 1$

   (b) $\frac{12}{3} = 4$ (e) $\frac{9}{9} = 1$

   (c) $\frac{0}{6} = 0$ (f) $\frac{0}{2} = 0$

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Mixed Numbers

(a) $5\frac{2}{3} = 5 + \frac{2}{3}$ (b) $1\frac{3}{8} = 1 + \frac{3}{8}$ (c) $15\frac{9}{11} = 15 + \frac{9}{11}$

Changing a Whole Number to an improper fraction

(a) $4 = \frac{4 \times 6}{1 \times 6} = \frac{24}{6}$

(b) $5 = \frac{5 \times 5}{1 \times 5} = \frac{25}{5}$

(c) $7 = \frac{7 \times 5}{1 \times 5} = \frac{35}{5}$

(d) $10 = \frac{10 \times 9}{1 \times 9} = \frac{90}{9}$

(e) $9 = \frac{9 \times 9}{1 \times 9} = \frac{81}{9}$

(f) $1 = \frac{1 \times 100}{1 \times 100} = \frac{100}{100}$

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2. (a) $1\frac{1}{2} = \frac{(1 \times 2) + 1}{2} = \frac{3}{2}$

(b) $5\frac{1}{8} = \frac{(5 \times 8) + 1}{8} = \frac{41}{8}$

(c) $7\frac{3}{5} = \frac{(7 \times 5) + 3}{5} = \frac{38}{5}$

(d) $4\frac{4}{5} = \frac{(4 \times 5) + 4}{5} = \frac{24}{5}$

(e) $10\frac{11}{12} = \frac{(10 \times 12) + 11}{12} = \frac{131}{12}$

4. Answers will vary from student to student – as long as the improper fractions can be simplified into whole numbers your answer will be acceptable.

Examples:

$\frac{12}{2} = 6$ $\frac{48}{2} = 24$ $\frac{81}{9} = 9$ $\frac{18}{3} = 6$

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5.

Fraction	Kind of Fraction	Whole Number (if any)
(a) $\frac{3}{8}$	proper	–
(b) $\frac{15}{16}$	proper	–
(c) $\frac{16}{4}$	improper	4
(d) $\frac{9}{22}$	proper	–
(e) $\frac{31}{31}$	improper	1
(f) $\frac{24}{6}$	improper	4
(g) $\frac{1}{2}$	proper	–
(h) $\frac{6}{7}$	proper	–
(i) $\frac{18}{6}$	improper	3

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(a) $\frac{85}{9} = 85 \div 9 = 9$ remainder $4 = 9\frac{4}{9}$

(b) $\frac{62}{7} = 62 \div 7 = 8$ remainder $6 = 8\frac{6}{7}$

(c) $\frac{36}{3} = 36 \div 3 = 12$ remainder $0 = 12$

(d) $\frac{12}{11} = 12 \div 11 = 1$ remainder $1 = 1\frac{1}{11}$

(e) $\frac{50}{5} = 50 \div 5 = 10$ remainder $0 = 10$

(f) $\frac{9}{8} = 9 \div 8 = 1$ remainder $1 = 1\frac{1}{8}$

(g) $\frac{121}{10} = 121 \div 10 = 12$ remainder $1 = 12\frac{1}{10}$

(h) $\frac{75}{19} = 75 \div 19 = 3$ remainder $18 = 3\frac{18}{19}$

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(a) $\frac{6}{8}$ inch

(b) $\frac{2}{4}$ inch and $\frac{4}{8}$ inch

(c) $\frac{2}{16}$ inch

(d) equivalent

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All the answers to the following questions will vary.

(a) $\frac{1}{2}$ $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ $\frac{1 \times 8}{2 \times 8} = \frac{8}{16}$

(b) $\frac{5}{4}$ $\frac{5 \times 3}{4 \times 3} = \frac{15}{12}$ $\frac{5 \times 9}{4 \times 9} = \frac{45}{36}$

(c) $\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$ $\frac{3 \times 17}{8 \times 17} = \frac{51}{136}$

(d) $\frac{3}{2}$ $\frac{3 \times 9}{2 \times 9} = \frac{27}{18}$ $\frac{3 \times 13}{2 \times 13} = \frac{39}{26}$

(e) $\frac{3}{4}$ $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{3 \times 15}{4 \times 15} = \frac{45}{60}$

(f) $\frac{5}{6}$ $\frac{5 \times 5}{6 \times 5} = \frac{25}{30}$ $\frac{5 \times 12}{6 \times 12} = \frac{60}{72}$

(g) $\frac{6}{1}$ $\frac{6 \times 7}{1 \times 7} = \frac{42}{7}$ $\frac{6 \times 37}{1 \times 37} = \frac{222}{37}$

(h) $\frac{1}{4}$ $\frac{1 \times 8}{4 \times 8} = \frac{8}{32}$ $\frac{1 \times 9}{4 \times 9} = \frac{9}{36}$

(i) $\frac{2}{3}$ $\frac{2 \times 11}{3 \times 11} = \frac{22}{33}$ $\frac{2 \times 16}{3 \times 16} = \frac{32}{48}$

(j) $\frac{8}{5}$ $\frac{8 \times 6}{5 \times 6} = \frac{48}{30}$ $\frac{8 \times 100}{5 \times 100} = \frac{800}{500}$

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(a) $\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$ (b) $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

(c) $\frac{10}{4} = \frac{10 \div 2}{4 \div 2} = \frac{5}{2}$ (d) $\frac{9}{6} = \frac{9 \div 3}{6 \div 3} = \frac{3}{2}$

(e) $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ (f) $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

(g) $\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$ (h) $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

B. Raising Fraction to Higher Terms

(a) $\frac{1}{2} = \frac{1 \times 8}{2 \times 8} = \frac{8}{16}$ (b) $\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$

(c) $\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$ (d) $\frac{7}{8} = \frac{7 \times 2}{8 \times 2} = \frac{14}{16}$

(e) $\frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$

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2. (b) $\frac{15}{4} = \frac{12}{4} + \frac{3}{4} = 3\frac{3}{4} = 3\frac{3 \times 8}{4 \times 8} = 3\frac{24}{32}$

(c) $\frac{17}{4} = \frac{16}{4} + \frac{1}{4} = 4\frac{1}{4} = 4\frac{1 \times 8}{4 \times 8} = 4\frac{8}{32}$

(d) $\frac{19}{8} = \frac{16}{8} + \frac{3}{8} = 2\frac{3}{8} = 2\frac{3 \times 4}{8 \times 4} = 2\frac{12}{32}$

(e) $\frac{72}{16} = \frac{64}{16} + \frac{8}{16} = 4\frac{8}{16} = 4\frac{8 \times 2}{16 \times 2} = 4\frac{16}{32}$

(f) $\frac{38}{8} = \frac{32}{8} + \frac{6}{8} = 4\frac{6}{8} = 4\frac{6 \times 4}{8 \times 4} = 4\frac{24}{32}$

3. (a) $\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}$

(b) $\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}$

(c) $\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \frac{30}{48}$

(d) $\frac{0}{2}, \frac{0}{4}, \frac{0}{6}, \frac{0}{8}, \frac{0}{10}, \frac{0}{12}$

(e) $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}$

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Exercise

(b) 27; factors are 1, 3, 9, 27.
(c) 16; factors are 1, 2, 4, 8, 16.
(d) 24; factors are 1, 2, 3, 4, 6, 8, 12, 24.
(e) 35; factors are 1, 5, 7, 35.
(f) 50; factors are 1, 2, 5, 10, 25, 50.

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(a) $15 = 3 \times 5$ (b) 11 prime (c) $20 = 2 \times 2 \times 5$
(d) $24 = 2 \times 2 \times 2 \times 3$ (e) $9 = 3 \times 3$ (f) 7 prime
(g) 29 prime (h) $32 = 2 \times 2 \times 2 \times 2 \times 2$ (i) $49 = 7 \times 7$
(j) $16 = 2 \times 2 \times 2 \times 2$ (k) $50 = 2 \times 5 \times 5$ (l) 23 prime

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Fraction	Prime Factors of Numerator and Denominator	Common Prime Factors	Equivalent Fractions (Lowest Terms)
$\frac{39}{65}$	$\frac{39}{65} = \frac{3 \times 13}{5 \times 13}$	13	$\frac{39 \div 13}{65 \div 13} = \frac{3}{5}$
$\frac{20}{30}$	$\frac{20}{30} = \frac{2 \times 2 \times 5}{2 \times 3 \times 5}$	2, 5	$\frac{20 \div 10}{30 \div 10} = \frac{2}{3}$
$\frac{56}{96}$	$\frac{56}{96} = \frac{2\times2\times2\times7}{2\times2\times2\times2\times2\times3}$	2, 2, 2	$\frac{56 \div 8}{96 \div 8} = \frac{7}{12}$
$\frac{24}{36}$	$\frac{24}{36} = \frac{2\times2\times2\times3}{2\times2\times3\times3}$	2, 2, 3	$\frac{24 \div 12}{36 \div 12} = \frac{2}{3}$
$\frac{75}{50}$	$\frac{75}{50} = \frac{3\times5\times5}{2\times5\times5}$	5, 5	$\frac{75 \div 25}{50 \div 25} = \frac{3}{2}$
$\frac{63}{56}$	$\frac{63}{56} = \frac{3\times3\times7}{2\times2\times2\times7}$	7	$\frac{63 \div 7}{56 \div 7} = \frac{9}{8}$

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2. (a) $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$

   (b) $\frac{3}{4}$

3. $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}\right\}$

   (a) $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}$

   (b) $\frac{8}{12}, \frac{10}{15}, \frac{12}{18}$ – Many other answers are acceptable.

   (c) $\frac{2}{3}$

4. (a) $\frac{4}{3} = \frac{4 \times 4}{3 \times 4} = \frac{16}{12}$

(b) $\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36}$

(c) $\frac{7}{4} = \frac{7 \times 4}{4 \times 4} = \frac{28}{16}$

(d) $\frac{10}{3} = \frac{10 \times 10}{3 \times 10} = \frac{100}{30}$

(e) $\frac{90}{60} = \frac{90 \div 30}{60 \div 30} = \frac{3}{2}$

(f) $\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

(g) $\frac{7}{8} = \frac{7 \times 18}{8 \times 18} = \frac{126}{144}$

(h) $\frac{7}{28} = \frac{7 \div 7}{28 \div 7} = \frac{1}{4}$

(i) $\frac{24}{64} = \frac{24 \div 8}{64 \div 8} = \frac{3}{8}$

(j) $\frac{49}{56} = \frac{49 \div 7}{56 \div 7} = \frac{7}{8}$

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1. (a) $\frac{14}{15}$ lowest

(b) $\frac{8}{12}$ ——— (not in lowest terms - lowest term would be $\frac{2}{3}$)

(c) $\frac{15}{28}$ lowest

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2. (a) $\frac{6}{10}$ ———

(b) $\frac{9}{15}$ ———

(c) $\frac{8}{15}$ lowest

(d) $\frac{10}{18}$ ———

(e) $\frac{9}{24}$ ———

(f) $\frac{15}{20}$ ———

(g) $\frac{10}{21}$ lowest

(h) $\frac{25}{30}$ ———

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Why did we divide 60 and 84 by 12?

12 is the highest common factor of 60 and 84.

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2. (a) $\frac{16}{28} = \frac{16 \div 4}{28 \div 4} = \frac{4}{7}$

(b) $\frac{5}{20} = \frac{5 \div 5}{20 \div 5} = \frac{1}{4}$

(c) $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

(d) $\frac{8}{9}$ = lowest terms

(e) $\frac{20}{12} = \frac{20 \div 4}{12 \div 4} = \frac{5}{3}$

(f) $\frac{16}{64} = \frac{16 \div 16}{64 \div 16} = \frac{1}{4}$

(g) $\frac{63}{81} = \frac{63 \div 9}{81 \div 9} = \frac{7}{9}$

(h) $\frac{8}{27}$ = lowest terms

(i) $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$

(j) $\frac{8}{36} = \frac{8 \div 4}{36 \div 4} = \frac{2}{9}$

3. (a) $\frac{3}{100} = \frac{3 \times 10}{100 \times 10} = \frac{30}{1000}$

(b) $\frac{7}{10} = \frac{7 \times 100}{10 \times 100} = \frac{700}{1000}$

(c) $\frac{47}{100} = \frac{47 \times 10}{100 \times 10} = \frac{470}{1000}$

(d) $\frac{100}{10\,000} = \frac{100 \div 10}{10\,000 \div 10} = \frac{10}{1000}$

Page 1

1. Answers will vary. (There are many acceptable answers.)

(a) $\frac{1}{8}$ $\frac{3}{8}, \frac{5}{8}, \frac{6}{8}$

(b) $\frac{2}{7}$ $\frac{3}{7}, \frac{4}{7}, \frac{1}{7}$

(c) $\frac{7}{3}$ $\frac{2}{3}, \frac{1}{3}, \frac{13}{3}$

(d) $\frac{5}{6}$ $\frac{1}{6}, \frac{3}{6}, \frac{27}{6}$

(e) $\frac{1}{2}$ $\frac{3}{2}, \frac{5}{2}, \frac{11}{2}$

(f) $\frac{9}{16}$ $\frac{1}{16}, \frac{3}{16}, \frac{5}{16}$

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(b) Change to 12ths

$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

$\frac{2}{4} = \frac{2 \times 3}{4 \times 3} = \frac{6}{12}$

(c) Change to 16ths

$\frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$

$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$

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(d) Change to 6ths

$\frac{5}{6} = \frac{5 \times 1}{6 \times 1} = \frac{5}{6}$

$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

(e) Change to 21sts

$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$

$\frac{1}{3} = \frac{1 \times 7}{3 \times 7} = \frac{7}{21}$

(f) Change to 10ths

$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$

$\frac{7}{10} = \frac{7 \times 1}{10 \times 1} = \frac{7}{10}$

(g) Change to 15ths

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$

$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$

(h) Change to 20ths

$\frac{9}{4} = \frac{9 \times 5}{4 \times 5} = \frac{45}{20}$

$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$

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1. (b) $\frac{6}{7} + \frac{5}{7} = \frac{6+5}{7} = \frac{11}{7} = 1\frac{4}{7}$ (c) $\frac{8}{3} + \frac{9}{3} = \frac{8+9}{3} = \frac{17}{3} = 5\frac{2}{3}$

(d) $\frac{6}{24} + \frac{6}{24} = \frac{6+6}{24} = \frac{12}{24} = \frac{1}{2}$ (e) $\frac{6}{8} + \frac{0}{8} = \frac{6+0}{8} = \frac{6}{8} = \frac{3}{4}$

(f) $\frac{18}{24} + \frac{6}{24} = \frac{18+6}{24} = \frac{24}{24} = 1$ (g) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1+2+5}{8} = \frac{8}{8} = 1$

(h) $\frac{5}{10} + \frac{7}{10} + \frac{2}{10} = \frac{5+7+2}{10} = \frac{14}{10} = \frac{7}{5} = 1\frac{2}{5}$

2. (b) $\frac{1}{3} + \frac{4}{3} = \frac{5}{3} = 1\frac{2}{3}$ (c) $\frac{5}{9} + \frac{2}{9} = \frac{7}{9}$ (d) $\frac{3}{16} + \frac{15}{16} = \frac{18}{16} = \frac{9}{8} = 1\frac{1}{8}$

Remember: When you add similar fractions (having thhe same denominator) you only have to add the numerators.

(e) $\frac{2}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$ (f) $\frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ (g) $\frac{1}{32} + \frac{15}{32} + \frac{9}{32} = \frac{25}{32}$ (h) $\frac{9}{5} + \frac{1}{5} + \frac{4}{5} = \frac{14}{5} = 2\frac{4}{5}$

3. The overall length would be:

$$\frac{25}{32} + \frac{17}{32} + \frac{29}{32} + \frac{15}{32} = \frac{25+17+29+15}{32} = \frac{86}{32} = \frac{86 \div 2}{32 \div 2} = \frac{43}{16} = 2\frac{11}{16} \text{ (inches)}$$

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1. (a) $3\frac{4}{7} + 5\frac{1}{7} = (3 + 5) + \frac{4}{7} + \frac{1}{7} = 8 + \frac{5}{7} = 8\frac{5}{7}$

(b) $2\frac{1}{5} + 2\frac{2}{5} = (2 + 2) + \frac{1}{5} + \frac{2}{5} = 4 + \frac{3}{5} = 4\frac{3}{5}$

(c) $7\frac{1}{2} + 3 = (7 + 3) + \frac{1}{2} = 10 + \frac{1}{2} = 10\frac{1}{2}$

(d) $8\frac{1}{8} + 4\frac{3}{8} = (8 + 4) + \frac{1}{8} + \frac{3}{8} = 12 + \frac{4}{8} = 12\frac{4}{8} = 12\frac{1}{2}$

(e) $9\frac{7}{12} + 4\frac{1}{12} = (9 + 4) + \frac{7}{12} + \frac{1}{12} = 13 + \frac{8}{12} = 13\frac{8}{12} = 13\frac{2}{3}$

(f) $3\frac{1}{8} + \frac{5}{8} = 3 + \left(\frac{1}{8} + \frac{5}{8}\right) = 3 + \frac{6}{8} = 3\frac{6}{8} = 3\frac{3}{4}$

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1. (a) $2\frac{1}{3} + \frac{2}{3} = 2 + \left(\frac{1}{3} + \frac{2}{3}\right) = 2 + \frac{3}{3} = 2 + 1 = 3$

(b) $5\frac{5}{9} + 6\frac{7}{9} = 11 + \left(\frac{5}{9} + \frac{7}{9}\right) = 11 + \frac{12}{9} = 11 + 1\frac{3}{9} = 12\frac{3}{9} = 12\frac{1}{3}$

(c) $3\frac{5}{7} + 1\frac{6}{7} = (3 + 1) + \left(\frac{5}{7} + \frac{6}{7}\right) = 4 + \frac{11}{7} = 4 + 1\frac{4}{7} = 5\frac{4}{7}$

(d) $7\frac{11}{12} + 2\frac{7}{12} = (7 + 2) + \left(\frac{11}{12} + \frac{7}{12}\right) = 9 + \frac{18}{12} = 9 + 1\frac{1}{2} = 10\frac{1}{2}$

(e) $3\frac{3}{10} + 2\frac{7}{10} = (3 + 2) + \left(\frac{3}{10} + \frac{7}{10}\right) = 5 + \frac{10}{10} = 5 + 1 = 6$

2. Length of screwdriver

$$2\frac{13}{16} + \frac{11}{16} + 1\frac{15}{16} = (2 + 1) + \left(\frac{13}{16} + \frac{11}{16} + \frac{15}{16}\right)$$

$$= 3 + \frac{39}{16}$$

$$= 3 + 2\frac{7}{16}$$

$$= 5\frac{7}{16}$$

The overall length of the screwdriver is $5\frac{7}{16}$ ".

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1. (b) $\frac{5}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$ (c) $\frac{9}{4} - \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$

   (d) $\frac{15}{16} - \frac{13}{16} = \frac{2}{16} = \frac{1}{8}$ (e) $\frac{7}{8} - \frac{5}{8} = \frac{2}{8} = \frac{1}{4}$

   (f) $\frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$ (g) $\frac{5}{6} - \frac{0}{6} = \frac{5}{6}$

   (h) $\frac{31}{32} - \frac{11}{32} = \frac{20}{32} = \frac{5}{8}$

2. $\frac{15}{16} - \frac{7}{16} = \frac{8}{16} = \frac{1}{2}$ The measure represented by 'a' is $\frac{1}{2}$.

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1. (a) $11\frac{5}{8} - 8\frac{3}{8} = (11 - 8) + \left(\frac{5}{8} - \frac{3}{8}\right) = 3 + \frac{2}{8} = 3\frac{1}{4}$

   (b) $1\frac{2}{5} - \frac{1}{5} = 1 + \left(\frac{2}{5} - \frac{1}{5}\right) = 1\frac{1}{5}$

   (c) $9\frac{3}{4} - \frac{1}{4} = 9 + \left(\frac{3}{4} - \frac{1}{4}\right) = 9\frac{2}{4} = 9\frac{1}{2}$

   (d) $7\frac{7}{8} - 3\frac{3}{8} = (7 - 3) + \left(\frac{7}{8} - \frac{3}{8}\right) = 4 + \frac{4}{8} = 4 + \frac{1}{2} = 4\frac{1}{2}$

   (e) $105\frac{31}{32} - 9\frac{1}{32} = (105 - 9) + \left(\frac{31}{32} - \frac{1}{32}\right) = 96 + \frac{30}{32} = 96\frac{15}{16}$

   (f) $21\frac{3}{4} - 7\frac{1}{4} = (21 - 7) + \left(\frac{3}{4} - \frac{1}{4}\right) = 14 + \frac{2}{4} = 14\frac{1}{2}$

   (g) $6\frac{3}{8} - 5\frac{3}{8} = (6 - 5) + \left(\frac{3}{8} - \frac{3}{8}\right) = 1 + \frac{0}{8} = 1$

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1. (a) $21\frac{1}{4} - 7\frac{3}{4}$

$= 20\frac{5}{4} - 7\frac{3}{4}$

$= (20 - 7) + \left(\frac{5}{4} - \frac{3}{4}\right)$

$= 13 + \frac{2}{4}$

$= 13\frac{2}{4}$

$= 13\frac{1}{2}$

(b) $6\frac{3}{8} - 5\frac{5}{8}$

$= 5\frac{11}{8} - 5\frac{5}{8}$

$= (5 - 5) + \left(\frac{11}{8} - \frac{5}{8}\right)$

$= 0 + \frac{6}{8}$

$= \frac{6}{8}$

$= \frac{3}{4}$

(c) $15 - 14\frac{7}{8}$

$= 14\frac{8}{8} - 14\frac{7}{8}$

$= \frac{1}{8}$

(d) $15\frac{7}{8} - 14$

$= (15 - 14) + (\frac{7}{8} - 0)$

$= 1 + \frac{7}{8} = 1\frac{7}{8}$

(e) $2\frac{5}{9} - 2\frac{4}{9}$

$= (2 - 2) + \frac{5}{9} - \frac{4}{9}$

$= 0 + \frac{1}{9} = \frac{1}{9}$

(f) $1\frac{1}{5} - \frac{4}{5}$

$= \frac{6}{5} - \frac{4}{5}$

$= \frac{2}{5}$

2. Weight of the Grease

$420\frac{3}{4} - 45\frac{1}{4}$

$= (420 - 45) + \left(\frac{3}{4} - \frac{1}{4}\right)$

$= 375 + \frac{2}{4}$

$= 375\frac{1}{2}$ The weight of the grease is $375\frac{1}{2}$ lbs.

3. How much smaller is the triangle from the rectangle?

$$27\frac{3}{8} - 15\frac{7}{8}$$

$$= 26\frac{11}{8} - 15\frac{7}{8}$$

$$= (26 - 15) + \left(\frac{11}{8} - \frac{7}{8}\right)$$

$$= 11 + \frac{4}{8}$$

$$= 11 + \frac{1}{2} = 11\frac{1}{2}$$ The rectangle is $11\frac{1}{2}$ square units smaller than the triangle.

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Exercises

(a) unlike
(b) like
(c) like
(d) like
(e) unlike
(f) unlike

B. Lowest Common Denominator

The like fractions are $\frac{3}{6}$ and $\frac{4}{6}$. The other like fractions are $\frac{6}{12}$ and $\frac{8}{12}$.

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(b) $\frac{3}{4}$ and $\frac{5}{6}$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

$$\frac{5}{6} = \frac{10}{12}$$ so $\frac{9}{12}$ and $\frac{10}{12}$ are like fractions.

(c) $\frac{1}{3}$ and $\frac{2}{9}$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$$

$$\frac{2}{9} = \frac{2}{9}$$ $\frac{2}{9}$ and $\frac{3}{9}$ are like fractions

(d) $\frac{1}{4}$ and $\frac{2}{5}$

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$

$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$ $\frac{5}{20}$ and $\frac{8}{20}$ are like fractions

(e) $\frac{1}{3}$ and $\frac{4}{5}$

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$

$\frac{4}{5} = \frac{8}{10} = \frac{12}{15}$ $\frac{5}{15}$ and $\frac{12}{15}$ are like fractions

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1.

(a)	2;	12, 16	Answers	(e)	4;	12, 36	Answers
(b)	7;	14, 28	will vary	(f)	9;	18, 27	will vary
(c)	5;	10, 15		(g)	11;	44, 33	
(d)	8;	40, 56		(h)	25;	50, 100	

2.

(a)	15, 120	Yes	(f)	7, 54	No
(b)	3, 81	Yes	(g)	11, 99	Yes
(c)	9, 18	Yes	(h)	25, 605	No
(d)	7, 24	No	(i)	16, 64	Yes
(e)	8, 56	Yes	(j)	10, 10 000	Yes

Bottom of Page 18

Given 5, 3 and 15. Are 15 and 30 both multiples of the numbers? **Yes.**

Is 15 or 30 the least common multiple? **15 is the L.C.M.**

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What is 19 divisible by? **1 and itself**
Is 29 a prime? **Yes**

Solution
$10 = 2 \times 5$
$15 = 3 \times 5$
$6 = 2 \times 3$
L.C.M. $= 2 \times 3 \times 5 = 30$ Find the product of the different primes.

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$4 = 2 \times 2$
$6 = 2 \times 3$
$8 = 2 \times 2 \times 2$
L.C.M. $= 2 \times 2 \times 2 \times 3 = 24$

The L.C.M. of $\frac{1}{4}$, $\frac{2}{6}$ and $\frac{5}{8}$ is 24.

Page 24

1. (a) $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

(b) $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

(c) $\frac{5}{12} + \frac{1}{4} = \frac{5}{12} = \frac{3}{12} = \frac{8}{12} = \frac{2}{3}$

(d) $\frac{1}{8} + \frac{1}{6} = \frac{3}{24} + \frac{4}{24} = \frac{7}{24}$

(e) $\frac{5}{6} = \frac{15}{18}$

$\frac{1}{9} = \frac{2}{18}$

$\frac{17}{18}$

(f) $\frac{3}{10} = \frac{6}{20}$

$\frac{1}{4} = \frac{5}{20}$

$\frac{11}{20}$

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3. (a) $\frac{3}{7} = \frac{15}{35}$

$\frac{1}{5} = \frac{7}{35}$

$\frac{22}{35}$

(b) $\frac{1}{8} = \frac{3}{24}$

$\frac{5}{6} = \frac{20}{24}$

$\frac{23}{24}$

(c) $\frac{1}{2} = \frac{9}{18}$

$\frac{1}{9} = \frac{2}{18}$

$\frac{11}{18}$

(d) $\frac{2}{5} = \frac{8}{20}$

$\frac{11}{20} = \frac{11}{20}$

$\frac{19}{20}$

(e) $\frac{72}{5} + \frac{7}{10} = \frac{72 \times 2}{5 \times 2} + \frac{7}{10} = \frac{144}{10} + \frac{7}{10} = \frac{151}{10} = 15\frac{1}{10}$

(f) $\frac{11}{36} + \frac{5}{24} = \frac{11 \times 2}{36 \times 2} + \frac{5 \times 3}{24 \times 3} = \frac{22}{72} + \frac{15}{72} = \frac{37}{72}$

(g) $\frac{5}{9} + \frac{3}{8} + \frac{2}{15} = \frac{5 \times 40}{9 \times 40} = \frac{3 \times 45}{8 \times 45} + \frac{2 \times 24}{15 \times 24} = \frac{200}{360} + \frac{135}{360} + \frac{48}{360} = \frac{383}{360}$

$= 1\frac{23}{360}$

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4. His whole complete salary can be expressed as 1. So the amount left will be

$$1 - \left(\frac{1}{4} + \frac{1}{3} + \frac{1}{6} + \frac{1}{8}\right)$$

$$= 1 - \left(\frac{6}{24} + \frac{8}{24} + \frac{4}{24} + \frac{3}{24}\right)$$

$$= 1 - \left(\frac{21}{24}\right)$$

$$= \frac{24}{24} - \frac{21}{24} = \frac{3}{24} = \frac{1}{8}$$

Conclusion: $\frac{1}{8}$ of Mr. James' salary is left for personal expenses and savings.

5. $x = \frac{47}{64} + \frac{5}{8} + \frac{31}{32} + \frac{11}{16}$

$$= \frac{47}{64} + \frac{5 \times 8}{8 \times 8} + \frac{31 \times 2}{32 \times 2} + \frac{11 \times 4}{16 \times 4}$$

$$= \frac{47}{64} + \frac{40}{64} + \frac{62}{64} + \frac{44}{64}$$

$$= \frac{193}{64}$$

$$= 3\frac{1}{64}$$

The overall length is $3\frac{1}{64}$ ".

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1. (b) $4\frac{1}{3} + 7\frac{2}{3} = (4 + 7) + \frac{1}{3} + \frac{2}{3} = 11 + \frac{3}{3} = 11 + 1 = 12$

(c) $2\frac{1}{4} + 5\frac{5}{6} = (2 + 5) + \left(\frac{3}{12} + \frac{10}{12}\right) = 7 + \frac{13}{12} = 7 + 1\frac{1}{12} = 8\frac{1}{12}$

(d) $1\frac{11}{12} + 8\frac{7}{20} = (1 + 8) + \left(\frac{55}{60} + \frac{21}{60}\right) = 9 + \frac{76}{60} = 9 + 1\frac{16}{60} = 10\frac{16}{60} = 10\frac{4}{15}$

(e) $2\frac{1}{2} + 3\frac{1}{11} = (2 + 3) + \left(\frac{1}{2} + \frac{1}{11}\right) = 5 + \left(\frac{11}{22} + \frac{2}{22}\right) = 5\frac{13}{22}$

(f) $15\frac{2}{3} + 23\frac{4}{7} + 35\frac{2}{5} = 15\frac{70}{105} + 23\frac{60}{105} + 35\frac{42}{105}$

$= 15 + 23 + 35 + \frac{172}{105}$

$= 73 + 1\frac{67}{105}$

$= 74\frac{67}{105}$

3. Thickness of Beam

$= 1\frac{1}{2} + 1\frac{1}{2} + 2\frac{5}{8} + 2\frac{5}{8} + 3\frac{3}{4}$

$= (1 + 1 + 2 + 2 + 3) + \frac{1}{2} + \frac{1}{2} + \frac{5}{8} + \frac{5}{8} + \frac{3}{4}$

$= 9 + \frac{4}{8} + \frac{4}{8} + \frac{5}{8} + \frac{5}{8} + \frac{6}{8}$

$= 9 + \frac{24}{8}$

$= 9 + 3 = 12''$

Conclusion: The laminated beam is 12 inches thick.

4. Total Distance

$= 117\frac{5}{6} + 120\frac{7}{8} + 105\frac{5}{16} + 110\frac{2}{3} + 115$

$= (117 + 120 + 105 + 110 + 115) + \frac{5}{16} + \frac{7}{8} + \frac{5}{16} + \frac{2}{3}$

$= 567 + \left(\frac{40}{48} + \frac{42}{48} + \frac{15}{48} + \frac{32}{48}\right)$

$$= 567 + \frac{129}{48}$$

$$= 567 + 2\frac{33}{48}$$

$$= 569\frac{33}{48} \qquad \left(\frac{33 \div 3}{48 \div 3} = \frac{11}{16}\right)$$

$$= 569\frac{11}{16}$$

The total distance travelled was $569\frac{11}{16}$ miles.

Page 29 **Exercise**

1. (a) $\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

(b) $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{3 \times 2}{4 \times 2} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}$

(c) $\frac{11}{16} - \frac{1}{2} = \frac{11}{16} - \frac{1 \times 8}{2 \times 8} = \frac{11}{16} - \frac{8}{16} = \frac{3}{16}$

(d) $\frac{2}{3} - \frac{1}{6} = \frac{2 \times 2}{3 \times 2} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

(e) $\frac{9}{10} - \frac{3}{5} = \frac{9}{10} - \frac{6}{10} = \frac{3}{10}$

(f) $\frac{8}{9} - \frac{1}{2} = \frac{8 \times 2}{9 \times 2} - \frac{1 \times 9}{2 \times 9} = \frac{16}{18} - \frac{9}{18} = \frac{7}{18}$

(g) $\frac{9}{10} = \frac{81}{90}$

$$-\frac{5}{9} = \frac{50}{90}$$

$$\frac{31}{90}$$

(h) $\frac{6}{5} = \frac{42}{35}$

$-\frac{3}{7} = \frac{15}{35}$

$\frac{27}{35}$

(i) $\frac{14}{5} = \frac{42}{15}$

$-\frac{7}{15} = \frac{7}{15}$

$\frac{35}{15} = \frac{7}{3} = 2\frac{1}{3}$

(j) $\frac{9}{11} = \frac{54}{66}$

$-\frac{1}{6} = \frac{11}{66}$

$\frac{43}{66}$

2. Interest on the Bond in the 3rd year would be

$$1 - \left(\frac{1}{3} + \frac{3}{5}\right) = 1 - \left(\frac{5}{15} + \frac{9}{15}\right)$$

$$= 1 - \frac{14}{15}$$

$$= \frac{15}{15} - \frac{14}{15}$$

$$= \frac{1}{15}$$

Conclusion: $\frac{1}{15}$th was paid the third year.

3. The two pipes will empty $\frac{1}{4} + \frac{7}{12}$ in one hour.

or $\frac{1}{4} + \frac{7}{12}$

$= \frac{3}{12} + \frac{7}{12} = \frac{10}{12} = \frac{5}{6}$

Conclusion: Both pipes will empty $\frac{5}{6}$ of the water in one hour.

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1. (b) $9\frac{4}{5} - 3\frac{13}{15} = 9\frac{12}{15} - 3\frac{13}{15} = 8\frac{27}{15} - 3\frac{13}{15} = 5\frac{14}{15}$

(c) $3\frac{4}{5} - 1\frac{2}{3} = 3\frac{12}{15} - 1\frac{10}{15} = 2\frac{2}{15}$

(d) $7\frac{3}{5} - \frac{3}{4} = 7\frac{12}{20} - \frac{15}{20} = 6\frac{32}{20} - \frac{15}{20} = 6\frac{17}{20}$

(e) $15\frac{1}{6} - \frac{3}{5} = 15\frac{5}{30} - \frac{18}{30} = 14\frac{35}{30} - \frac{18}{30} = 4\frac{17}{30}$

(f) $22\frac{3}{4} - 11\frac{5}{6} = 22\frac{18}{24} - 11\frac{20}{24} - 11\frac{20}{24} = 21\frac{42}{24} - 11\frac{20}{24} = 10\frac{22}{24} = 10\frac{11}{12}$

(g) $9 - \frac{3}{13} = 8\frac{13}{13} - \frac{3}{13} = 8\frac{10}{13}$

(h) $5 - \frac{1}{5} = 4\frac{5}{5} - \frac{1}{5} = 4\frac{4}{5}$

(i) $5\frac{7}{12} - \frac{13}{15} = 5\frac{35}{60} - \frac{52}{60} = 4\frac{95}{60} - \frac{52}{60} = 4\frac{43}{60}$

2. (a) $3\frac{1}{4} = 3\frac{2}{8} = 2\frac{10}{8}$

$-1\frac{5}{8} = 1\frac{5}{8} = 1\frac{5}{8}$

$1\frac{5}{8}$

(b) $8\frac{5}{12} = 8\frac{10}{24} = 7\frac{34}{24}$

$-2\frac{7}{8} = 2\frac{21}{24} = -2\frac{21}{24}$

$5\frac{13}{24}$

3. Material Machined Off

$= 8\frac{5}{16} - 7\frac{5}{8}$

$= 8\frac{5}{16} - 7\frac{10}{16}$

$= 7\frac{21}{16} - 7\frac{10}{16} = \frac{11}{16}$

Conclusion: $\frac{11}{16}$ lbs. of the material was machined off.

4. Amount of Board left would be

$16 - \left(2\frac{1}{2} + 3\frac{1}{4} + 1\frac{7}{16} + 1\frac{5}{8}\right)$

$= 16 - \left(2\frac{8}{16} + 3\frac{4}{16} + 1\frac{7}{16} + 1\frac{10}{16}\right)$

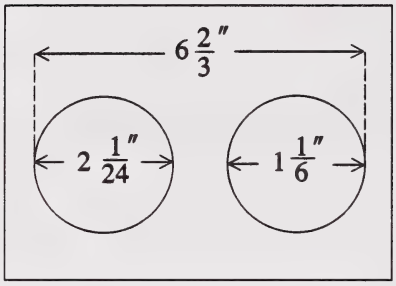
$= 16 - \left(7\frac{29}{16}\right)$

$= 16 - 8\frac{13}{16}$

$= 15\frac{16}{16} - 8\frac{13}{16} = 7\frac{3}{16}$

Conclusion: There are $7\frac{3}{16}$ feet left.

5. Hint – in a question like this, it is always worthwhile to draw a diagram to help you to visualiz the problem.



Length of Metal between circles.

$$6\frac{2}{3} - \left(2\frac{1}{24} + 1\frac{1}{6}\right)$$

$$= 6\frac{2}{3} - \left(2\frac{1}{24} + 1\frac{4}{24}\right)$$

$$= 6\frac{2}{3} - \left(3\frac{5}{24}\right)$$

$$= 6\frac{16}{24} - 3\frac{5}{24} = 3\frac{11}{24}$$

Conclusion: The length of the metal between the holes is $3\frac{11}{24}$ inches.

6. Amount of highway left to construct would be

Total amount of highway less amount already constructed.

$$= 15\frac{3}{8} - \left(3\frac{1}{2} + 5\frac{3}{4} + 2\frac{3}{16}\right)$$

$$= 15\frac{3}{8} - \left(3\frac{8}{16} + 5\frac{12}{16} + 2\frac{3}{16}\right)$$

$$= 15\frac{3}{8} - \left(10\frac{23}{16}\right)$$

$$= 15\frac{3}{8} - 11\frac{7}{16}$$

$$= 15\frac{6}{16} - 11\frac{7}{16}$$

$$= 14\frac{22}{16} - 11\frac{7}{16}$$

$$= 3\frac{15}{16}$$

Conclusion: $3\frac{15}{16}$ meters of highway remained uncompleted.

Page 1 **Exercise**

(a) $\frac{3}{4}$''

(b) 3''

(c) $\frac{6}{8}$'' or $\frac{3}{4}$''

(d) $\frac{4}{4}$'' or 1''

(e) $\frac{12}{4}$'' or 3''

(f) $\frac{3}{4}$''

(g) 1''

(h) $\frac{2}{16}$'' or $\frac{1}{8}$''

(i) $\frac{3}{8}$''

(j) $\frac{20}{8}$'' or $2\frac{4}{8}$'' or $2\frac{1}{2}$

Page 4

Number of kilograms of copper $= \frac{24}{29} \times 157 = \frac{3768}{29}$ kg or $129\frac{27}{29}$ kg.

Number of kilograms of tin $= \frac{4}{29} \times \frac{157}{1} = \frac{628}{29} = 21\frac{19}{29}$ kg.

Number of kilograms of zinc $= \frac{1}{29} \times \frac{157}{1} = \frac{157}{29} = 5\frac{12}{29}$ kg.

Page 5

(b) $\frac{8}{12} = \frac{4 \times 2}{4 \times 3} = \frac{2}{3}$

(c) $\frac{4}{16} = \frac{1 \times 4}{4 \times 4} = \frac{1}{4}$

(d) $\frac{5}{20} = \frac{1 \times 5}{4 \times 5} = \frac{1}{4}$

(e) $\frac{18}{24} = \frac{6 \times 3}{6 \times 4} = \frac{3}{4}$

(f) $\frac{5}{10} = \frac{5 \times 1}{5 \times 2} = \frac{1}{2}$

(g) $\frac{7}{21} = \frac{7 \times 1}{7 \times 3} = \frac{1}{3}$

(h) $\frac{5}{25} = \frac{5 \times 1}{5 \times 5} = \frac{1}{5}$

(i) $\frac{9}{27} = \frac{1 \times 9}{3 \times 9} = \frac{1}{3}$

(j) $\frac{16}{24} = \frac{8 \times 2}{8 \times 3} = \frac{2}{3}$

Page 7

1. (b) $\frac{5}{12} \times \frac{3}{10} = \frac{5 \times 3}{12 \times 10} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

(c) $\frac{6}{5} \times \frac{15}{18} = \frac{6}{5} \times \frac{14}{18} = \frac{1 \times 1}{1 \times 1} = \frac{1}{1} = 1$

(d) $\frac{3}{8} \times \frac{8}{9} = \frac{3}{8} \times \frac{8}{9} = \frac{1 \times 1}{1 \times 3} = \frac{1}{3}$

(e) $\frac{5}{6} \times \frac{12}{5} = \frac{5}{6} \times \frac{12}{5} = \frac{1 \times 2}{1 \times 1} = \frac{2}{1} = 2$

(f) $\frac{3}{4} \times \frac{20}{27} = \frac{3}{4} \times \frac{20}{27} = \frac{1 \times 5}{1 \times 9} = \frac{5}{9}$

(g) $\frac{6}{9} \times \frac{18}{36} = \frac{6}{9} \times \frac{18}{36} = \frac{2}{6} = \frac{1}{3}$

(h) $\frac{5}{8} \times \frac{6}{15} = \frac{5}{8} \times \frac{6}{15} = \frac{1 \times 1}{4 \times 1} = \frac{1}{4}$

(i) $\frac{4}{15} \times \frac{10}{28} = \frac{1 \times 2}{3 \times 7} = \frac{2}{21}$

(j) $\frac{7}{12} \times \frac{8}{21} = \frac{7}{12} \times \frac{8}{21} = \frac{2}{9}$

(k) $\frac{9}{10} \times \frac{25}{21} = \frac{9}{10} \times \frac{25}{21} = \frac{15}{14} = 1\frac{1}{14}$

(l) $\frac{7}{8} \times \frac{12}{14} = \frac{7}{8} \times \frac{12}{14} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$

Page 8

2. (a) $24 \times \frac{5}{24} = \frac{24}{1} \times \frac{5}{24} = \frac{5}{1} = 5$

(b) $10 \times \frac{3}{10} = \frac{10}{1} \times \frac{3}{10} = \frac{3}{1} = 3$

(c) $100 \times \frac{3}{100} = \frac{100}{1} \times \frac{3}{100} = \frac{3}{1} = 3$

(d) $100 \times \frac{3}{10} = \frac{100}{1} \times \frac{3}{10} = \frac{30}{1} = 30$

(e) $10 \times \frac{7}{100} = \frac{10}{1} \times \frac{7}{100} = \frac{7}{10}$

(f) $\frac{9}{10} \times \frac{20}{100} = \frac{9}{10} \times \frac{20}{100} = \frac{18}{100} = \frac{9}{50}$

(g) $24 \times \frac{5}{6} = \frac{24}{1} \times \frac{5}{6} = \frac{20}{1} = 20$

(h) $\frac{9}{15} \times \frac{7}{9} = \frac{9}{15} \times \frac{7}{9} = \frac{7 \times 1}{15 \times 1} = \frac{7}{15}$

(i) $\frac{7}{10} \times \frac{100}{21} \times \frac{9}{10} = \frac{7}{10} \times \frac{100}{21} \times \frac{9}{10} = \frac{1 \times 1 \times 3}{1 \times 1 \times 1} = \frac{3}{1} = 3$

(j) $\frac{5}{7} \times 49 \times \frac{2}{3} = \frac{5}{7} \times \frac{49}{1} \times \frac{2}{5} = \frac{1 \times 7 \times 2}{1 \times 1 \times 1} = \frac{14}{1} = 14$

3. The man's share is $\frac{3}{4} \times 960\,000 = \frac{3}{4} \times 960\,000 = \frac{3}{1} \times 240\,000$

$= \$720\,000$

So the man's share is \$720 000.

The value of his gift to the son is $\frac{7}{16} \times 720\,000$

$= \frac{7}{16} \times 720\,000$

$\frac{7}{1} \times 45\,000 = \$315\,000$

Conclusion: In dollars the value of the gift to his son is \$315 000.

Page 9

Give the multiplicative inverse of each of the following.

(a) $\frac{8}{1}$

(b) $\frac{7}{2}$

(c) $\frac{4}{5}$

(d) $\frac{1}{9}$

(e) $\frac{3}{10}$

(f) $\frac{100}{31}$

(g) $\frac{7}{11}$

(h) $\frac{2}{1}$

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1. (a) 1
   (b) 1
   (c) 1
   (d) 1
   (e) 1
   (f) 1

2. (a) $\frac{1}{10} \times \frac{10}{1} = 1$ The multiplicative inverse of $\frac{1}{10}$ is $\frac{10}{1}$ (or 10).

(b) $\frac{2}{7} \times \frac{7}{2} = 1$ The multiplicative inverse of $\frac{2}{7}$ is $\frac{7}{2}$.

(c) $7 \times \frac{1}{7} = 1$ The multiplicative inverse of 7 is $\frac{1}{7}$.

(d) $\frac{7}{6} \times \frac{6}{7} = 1$ The multiplicative inverse of $\frac{7}{6}$ is $\frac{6}{7}$.

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2. Number of gallons of antifreeze.

$$\frac{7}{16} \times 5\frac{1}{4}$$

$$= \frac{7}{16} \times \frac{21}{4} = \frac{147}{64} = 2\frac{19}{64}$$

There are $2\frac{19}{64}$ gallons of antifreeze.

3. Number of feet would be

$$3\frac{1}{2} \times 16\frac{1}{2}$$

$$\frac{7}{2} \times \frac{33}{2} = \frac{7 \times 33}{2 \times 2} = \frac{231}{4} = 57\frac{3}{4}$$

There are $57\frac{3}{4}$ feet in $3\frac{1}{2}$ rods.

4. The distance that the plane will travel is

$$620 \times 4\frac{3}{4} = \frac{620}{1} \times \frac{19}{4}$$

$$= \frac{620}{1} \times \frac{19}{4} = \frac{155 \times 19}{1 \times 1} = \frac{2945}{1} = 2945$$

The plane will travel 2945 miles.

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1. $6 \div \frac{3}{8} = 16$ (by count)

2. $5 \div \frac{1}{4} = 20$

3. $3 \div \frac{3}{16} = 16$

4. $4 \div \frac{1}{8} = 32 \left[4 \div \frac{1}{8} = 4 \times \frac{8}{1} = \frac{32}{1} = 32\right]$

5. $6 \div \frac{3}{16} = 32 \left[6 \div \frac{3}{16} = 6 \times \frac{16}{3} = \frac{32}{1} = 32\right]$

6. $2 \div \frac{1}{16} = 32 \left[2 \div \frac{1}{16} = 2 \times \frac{16}{1} = \frac{32}{1} = 32\right]$

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2. (a) $\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{2} \times \frac{1}{1} = \frac{3}{2} = 1\frac{1}{2}$

(b) $\frac{\frac{8}{1}}{\frac{1}{2}} = \frac{8}{1} \div \frac{1}{2} = \frac{8}{1} \times \frac{2}{1} = \frac{16}{1} = 16$

(c) $\frac{\frac{5}{16}}{\frac{3}{8}} = \frac{5}{16} \div \frac{3}{8} = \frac{5}{16} \times \frac{8}{3} = \frac{5}{16} \times \frac{8}{3} = \frac{5}{2} \times \frac{1}{3} = \frac{5}{6}$

(d) $\frac{\frac{1}{1}}{\frac{1}{8}} = 1 \div \frac{1}{8} = \frac{1}{1} \times \frac{8}{1} = \frac{8}{1} = 8$

(e) $\frac{\frac{3}{4}}{\frac{9}{10}} = \frac{3}{4} \div \frac{9}{10} = \frac{3}{4} \times \frac{10}{9} = \frac{3}{4} \times \frac{10}{9} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$

3. 8 feet = _____ inches.

$8 \times 12 = 96$ inches

There are 96 inches in 8 feet.

How many boards $\frac{3}{4}$'' thick would be in this pile?

$$96 \div \frac{3}{4} = \frac{96}{1} \times \frac{4}{3} = \frac{96}{1} \times \frac{4}{3} = \frac{32 \times 4}{1 \times 1} = \frac{128}{1} = 128$$

There are 128 boards.

4. The number of required turns can be found by calculating $3 \div \frac{1}{64}$.

$$3 \div \frac{1}{64} = 3 \times \frac{64}{1} = \frac{192}{1} = 192$$

192 turns are required.

Page 16

1. (b) $1\frac{2}{3} \div \frac{1}{4} = \frac{5}{3} \div \frac{1}{4} = \frac{5}{3} \times \frac{4}{1} = \frac{20}{3} = 6\frac{2}{3}$

(c) $7\frac{3}{4} \div 2\frac{1}{2} = \frac{31}{4} \div \frac{5}{2} = \frac{31}{4} \times \frac{2}{5} = \frac{31}{4} \times \frac{2}{5} = \frac{31}{10} = 3\frac{1}{10}$

(d) $18 \div 2\frac{2}{5} = \frac{18}{1} \div \frac{12}{5} = \frac{18}{1} \times \frac{5}{12} = \frac{18}{1} \times \frac{5}{12} = \frac{15}{2} = 7\frac{1}{2}$

(e) $25\frac{1}{2} \div 1\frac{1}{3} = \frac{51}{2} \div \frac{4}{3} = \frac{51}{2} \times \frac{3}{4} = \frac{51 \times 3}{2 \times 4} = \frac{153}{8} = 19\frac{1}{8}$

(f) $2\frac{9}{16} \div \frac{3}{8} = \frac{41}{16} \div \frac{3}{8} = \frac{41}{16} \times \frac{8}{3} = \frac{41}{16} \times \frac{8}{3} = \frac{41 \times 1}{2 \times 3} = \frac{41}{6} = 6\frac{5}{6}$

(g) $8\frac{3}{5} \div 6\frac{1}{7} = \frac{43}{5} \div \frac{43}{7} = \frac{43}{5} \times \frac{7}{43} = \frac{43}{5} \times \frac{7}{43} = \frac{1 \times 7}{5 \times 1} = \frac{7}{5} = 1\frac{2}{5}$

(h) $1\frac{1}{8} \div 6\frac{2}{5} = \frac{9}{8} \div \frac{32}{5} = \frac{9}{8} \times \frac{5}{32} = \frac{45}{256}$

(i) $3\frac{1}{3} \div 2\frac{1}{2} = \frac{10}{3} \div \frac{5}{2} = \frac{10}{3} \times \frac{2}{5} = \frac{10}{3} \times \frac{2}{5} = \frac{4}{3} = 1\frac{1}{3}$

(j) $42\frac{1}{3} \div 16\frac{1}{3} = \frac{127}{3} \div \frac{49}{3} = \frac{127}{3} \times \frac{3}{49} = \frac{127}{3} \times \frac{3}{49} = \frac{127}{49} = 2\frac{29}{49}$

(k) $3\frac{1}{4} \div \frac{13}{7} = \frac{13}{4} \div \frac{13}{7} = \frac{13}{4} \times \frac{7}{13} = \frac{13}{4} \times \frac{7}{13} = \frac{7}{4} = 1\frac{3}{4}$

2. (b) $15 \div 1\frac{1}{4} \times \frac{7}{2}$

$= 15 \div \frac{5}{4} \times \frac{7}{2}$

$= 15 \times \frac{4}{5} \times \frac{7}{2}$

$= \frac{3 \times 2 \times 7}{1 \times 1}$

$= \frac{42}{1} = 42$

(c) $\frac{5}{8} \div 10 \times \frac{2}{3}$

$= \frac{5}{8} \div \frac{10}{1} \times \frac{2}{3}$

$= \frac{5}{8} \times \frac{1}{10} \times \frac{2}{3}$

$= \frac{5}{8} \times \frac{1}{10} \times \frac{2}{3}$

$= \frac{1}{4} \times \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{24}$

(d) $1\frac{1}{48} \times 3\frac{1}{21} \div 4\frac{2}{3}$

$= \frac{49}{48} \times \frac{64}{21} \div \frac{14}{3}$

$= \frac{49}{48} \times \frac{64}{21} \times \frac{3}{14}$

$= \frac{49}{48} \times \frac{64}{21} \times \frac{3}{14}$

$= \frac{1 \times 8 \times 1}{\cancel{6 \times 1 \times 2}} = \frac{8}{12} = \frac{2}{3}$

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3. Number of blocks.

$35 \div 1\frac{3}{4}$

$= 35 \div \frac{7}{4}$

$= \frac{35}{1} \times \frac{4}{7}$

$= \frac{35}{1} \times \frac{4}{7} = \frac{5 \times 4}{1 \times 1} = \frac{20}{1} = 20$

Conclusion: 20 blocks of $1\frac{3}{4}$'' each can be cut from a 35'' long stick.

4. Length of each part $= 3\frac{1}{2} \div 4 \left(\text{not } 4 \div 3\frac{1}{2}\right)$

$= \frac{7}{2} \div \frac{4}{1}$

$= \frac{7}{2} \times \frac{1}{4} = \frac{7}{8}$

Conclusion: Each part is $\frac{7}{8}$ inches long.

5. Width of each slip would be $40\frac{1}{2} \div 4$

$40\frac{1}{2} \div \frac{4}{1}$

$\frac{81}{2} \times \frac{1}{4} = \frac{81}{8} = 10\frac{1}{8}$

Conclusion: The width of each slip is $10\frac{1}{8}$ inches.

6. Number of boards $= 49\frac{1}{2} \div 2\frac{1}{4}$

$= \frac{99}{2} \div \frac{9}{4}$

$= \frac{99}{2} \times \frac{4}{9} = \frac{11}{1} \times \frac{2}{1} = \frac{22}{1} = 22$

Conclusion: 22 boards will be needed to fit the floor space.

Page 20 **Optional Exercises**

(a) $\dfrac{\frac{9}{8}}{3} = \frac{9}{8} \div \frac{3}{1} = \frac{9}{8} \times \frac{1}{3} = \frac{9}{8} \times \frac{1}{3} = \frac{3}{8} \times \frac{1}{1} = \frac{3}{8}$

(b) $\dfrac{\frac{1}{4}}{\frac{2}{3}} = \frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$

(c) $\dfrac{\frac{4}{1}}{\frac{1}{5}} = 4 \div \frac{1}{5} = \frac{4}{1} \times \frac{5}{1} = \frac{20}{1} = 20$

(d) $\dfrac{\frac{5}{9}}{\frac{9}{5}} = \frac{5}{9} \div \frac{9}{5} = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$

(e) $\dfrac{\frac{1}{2}}{\frac{9}{10}} = \frac{1}{2} \div \frac{9}{10} = \frac{1}{2} \times \frac{10}{9} = \frac{1}{2} \times \frac{10}{9} = \frac{5}{9}$

(f) $\dfrac{1\frac{1}{2}}{3} = 1\frac{1}{2} \div 3 = \frac{3}{2} \div \frac{3}{1} = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$

(g) $\dfrac{2\frac{1}{4}}{1\frac{3}{8}} = 2\frac{1}{4} \div 1\frac{3}{8} = \frac{9}{4} \div \frac{11}{8} = \frac{9}{4} \times \frac{8}{11} = \frac{18}{11} = 1\frac{7}{11}$

(h) $\dfrac{1\frac{1}{9}}{2\frac{1}{2}} = 1\frac{1}{9} \div 2\frac{1}{2} = \frac{10}{9} \div \frac{5}{2} = \frac{10}{9} \times \frac{2}{5} = \frac{10}{9} \times \frac{2}{5} = \frac{4}{9}$

(i) $\dfrac{4\frac{1}{4}}{5\frac{4}{5}} = 4\frac{1}{4} \div 5\frac{4}{5} = \frac{17}{4} \div \frac{29}{5} = \frac{17}{4} \times \frac{5}{29} = \frac{17}{4} \times \frac{5}{29} = \frac{85}{116}$

Page 22

(a) $\dfrac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{16}}$

$= \dfrac{\frac{1}{8}}{\frac{1}{16}}$

$= \frac{1}{8} \div \frac{1}{16}$

$= \frac{1}{8} \times \frac{16}{1}$

$= \frac{2}{1}$

$= 2$

(b) $\dfrac{\frac{2}{5} + \frac{1}{4}}{\frac{1}{2} + \frac{4}{5}}$

$= \dfrac{\frac{8}{20} + \frac{5}{20}}{\frac{5}{10} + \frac{8}{10}}$

$= \dfrac{\frac{13}{20}}{\frac{13}{10}}$

$= \frac{13}{20} \div \frac{13}{10}$

$= \frac{13}{20} \times \frac{10}{13} = \frac{13}{20} \times \frac{10}{13} = \frac{1}{2}$

(c) $\dfrac{\frac{1}{8}+\frac{1}{5}}{\frac{3}{4}-\frac{2}{5}}$

$= \dfrac{\frac{5}{40}+\frac{24}{40}}{\frac{15}{20}-\frac{8}{20}}$

$= \dfrac{\frac{29}{40}}{\frac{7}{20}}$

$= \frac{29}{40} \div \frac{7}{20}$

$= \frac{29}{40} \times \frac{20}{7} = \frac{29}{14}$

$= 2\frac{1}{14}$

(d) $\dfrac{\frac{1}{6}}{\frac{1}{2}-\frac{1}{3}}$

$= \dfrac{\frac{5}{6}}{\frac{3}{6}-\frac{2}{6}}$

$= \dfrac{\frac{5}{6}}{\frac{1}{6}}$

$= \frac{5}{6} \div \frac{1}{6}$

$= \frac{5}{6} \times \frac{6}{1} = \frac{5}{6} \times \frac{6}{1} = \frac{5}{1}$

$= 5$

(e) $\dfrac{3 + \left(\frac{1}{2} \times 4\right)}{2 - 1\frac{1}{2}}$

$= \dfrac{3+2}{1\frac{2}{2}-1\frac{1}{2}}$

$= \dfrac{\frac{5}{1}}{\frac{1}{2}}$

$= 5 \div \frac{1}{2}$

$= \frac{5}{1} \times \frac{2}{1} = \frac{10}{1}$

$= 10$

(f) $\dfrac{\left(3\frac{1}{6} + 2\frac{5}{6}\right) \div \frac{2}{3}}{\frac{1}{2} \times 3\frac{1}{4} \div 6\frac{1}{2}}$

$= \dfrac{6 \div \frac{2}{3}}{\frac{1}{2} \times \frac{13}{4} \times \frac{2}{13}}$

$= \dfrac{\frac{6}{1} \times \frac{3}{2}}{\frac{1}{2} \times \frac{13}{4} \times \frac{2}{13}} = \dfrac{\frac{9}{1}}{\frac{1}{4}}$

$= \frac{9}{1} \div \frac{1}{4}$

$= \frac{9}{1} \times \frac{4}{1} = \frac{36}{1}$

$= 36$

(g) $$\frac{4 \times 3\frac{1}{7} \times \frac{2}{3}}{3 \times \frac{2}{3} \div 1\frac{1}{3}}$$

$$= \frac{\frac{4}{1} \times \frac{22}{7} \times \frac{2}{3}}{3 \times \frac{2}{3} \div \frac{4}{3}}$$

$$= \frac{\frac{4}{1} \times \frac{22}{7} \times \frac{2}{3}}{\frac{3}{1} \times \frac{2}{3} \times \frac{3}{4}} = \frac{\frac{176}{21}}{\frac{3}{2}}$$

$$= \frac{176}{21} \div \frac{3}{2} = \frac{176}{21} \times \frac{2}{3} = \frac{356}{63}$$

$$= 5\frac{37}{63}$$

(h) $$1 + \frac{1}{\frac{2}{3} + \frac{2}{5}}$$

$$= 1 + \frac{1}{\frac{10}{15} + \frac{6}{15}}$$

$$= 1 + \frac{1}{\frac{16}{15}}$$

$$= 1 + 1 \div \frac{16}{15}$$

$$= 1 + \frac{1}{1} \times \frac{15}{16}$$

$$= 1 + \frac{15}{16} = 1\frac{15}{16}$$

(i) $$\frac{8 \times \frac{9}{8}}{1 - \left(\frac{2}{3} - \frac{1}{4}\right)}$$

$$= \frac{\frac{9}{1}}{1 - \left(\frac{8}{12} - \frac{3}{12}\right)}$$

$$= \frac{\frac{9}{1}}{1 - \frac{5}{12}} = \frac{\frac{9}{1}}{\frac{12}{12} - \frac{5}{12}}$$

$$= \frac{\frac{9}{1}}{\frac{7}{12}} = \frac{9}{1} \div \frac{7}{12}$$

$$= \frac{9}{1} \times \frac{12}{7} = \frac{108}{7}$$

$$= 15\frac{3}{7}$$

(j) $$\frac{6 + 7\frac{1}{2}}{5 \div \frac{5}{8}} = \frac{13\frac{1}{2}}{\frac{5}{1} \times \frac{8}{5}}$$

$$= \frac{13\frac{1}{2}}{\frac{8}{1}} = 13\frac{1}{2} \div \frac{8}{1}$$

$$= \frac{27}{2} \div \frac{8}{1} = \frac{27}{2} \times \frac{1}{8} = \frac{27}{16}$$

$$= 1\frac{11}{16}$$

Page 26 **Problem 1**

Let "x" equal the length of the bar.

$$x = 9\frac{1}{2} \div 1\frac{7}{8}$$

$$x = \frac{19}{2} \div \frac{15}{8}$$

$$x = \frac{19}{2} \times \frac{8}{15} = \frac{19}{2} \times \frac{8}{15} = \frac{19}{1} \times \frac{4}{15} = \frac{76}{15}$$

$$x = \frac{76}{15} = 5\frac{1}{15} \text{ feet}$$

Conclusion: The length of the bar is $5\frac{1}{15}$ feet.

Problem 2

$$n = \frac{48\frac{1}{2} - 4\frac{1}{4}}{\frac{1}{8}}$$

$$n = \frac{48\frac{2}{4} - 4\frac{1}{4}}{\frac{1}{8}}$$

$$n = \frac{44\frac{1}{4}}{\frac{1}{8}}$$

$$n = 44\frac{1}{4} \div \frac{1}{8}$$

$$n = \frac{177}{4} \div \frac{1}{8}$$

$$n = \frac{177}{4} \times \frac{8}{1}$$

$$n = \frac{177}{4} \times \frac{8}{1} = \frac{177}{1} \times \frac{2}{1} = \frac{354}{1} = 354$$

Conclusion: There are 354 bolts in the box.

Page 2

(a)	17 more than 20	$20 + 17$
(b)	the difference between 5 and 2	$5 - 2$
(c)	the product of 10 and 11	10×11
(d)	one half of 18	$18 \div 2$ or $\frac{1}{2} \times 18$
(e)	17 over 6	$\frac{17}{6}$
(f)	15 less than 20	$20 - 15$
(g)	19 increased by 1	$19 + 1$
(h)	$\frac{1}{4}$ divided by $\frac{1}{2}$	$\frac{1}{4} \div \frac{1}{2}$
(i)	$\frac{7}{8}$ more than 5	$5 + \frac{7}{8}$
(j)	$\frac{1}{2}$ is diminished by $\frac{3}{8}$	$\frac{1}{2} - \frac{3}{8}$
(j)	$\frac{1}{2}$ is diminished by $\frac{3}{8}$	$\frac{1}{2} - \frac{3}{8}$
(k)	the quotient of 2 and $\frac{1}{3}$	$2 \div \frac{1}{3}$

Exercise 2

(a) $\frac{7}{8} + \frac{1}{2} = x$

(b) $8 + \frac{2}{3} = \frac{3}{4} \times x$

(c) $9 \div \frac{1}{9} = x$

(d) $x = 17 - 8\frac{1}{2}$

(e) $\frac{1}{2} \times x = 2\frac{1}{2}$

(f) $5 + 15 = x$

(g) $100 - 19 = x$

(h) $\frac{5}{8} \times 50 = x$

(i) $x - \frac{1}{4} = y$ (answers will vary)

(j) $10 \times 15 = x$

Page 4 **Problem**

The distance $A = \frac{9}{16} + \frac{1}{2} + 1\frac{1}{4}$ (You must find a common denominator)

$$= \frac{9}{16} + \frac{8}{16} + 1\frac{4}{16} = 1\frac{21}{16} = 1 + 1 + \frac{5}{16} = 2\frac{5}{16}$$

The overall length is $2\frac{5}{16}$ inches.

Page 5 **First Problem**

$A = 2\frac{7}{16} - \frac{3}{8}$

$A = 2\frac{7}{16} - \frac{6}{16}$

$A = 2\frac{1}{16}$

The distance A is $2\frac{1}{16}$ inches.

Second Problem

$$L = 7\frac{1}{8} - \left(\frac{7}{8} + 2\frac{1}{2}\right)$$

$$L = 7\frac{1}{8} - \left(\frac{7}{8} + 2\frac{4}{8}\right)$$

$$L = 7\frac{1}{8} - \left(2\frac{11}{8}\right)$$

$$L = 7\frac{1}{8} - 3\frac{3}{8}$$

$$L = 6\frac{9}{8} - 3\frac{3}{8}$$

$$L = 3\frac{6}{8}$$

$$L = 3\frac{3}{4}$$

The unknown length is $3\frac{3}{4}$ inches.

Page 6 **Problem**

Total length = 15 × 12
= 180 feet

Final Statement: The total length of the lumber needed is 180 feet.

Page 7 **Pulley Problem**

Width of each step is $4\frac{7}{8} \div 3$

$$= \frac{39}{8} \div \frac{3}{1}$$

$$= \frac{39}{8} \times \frac{1}{3} = \frac{39}{8} \times \frac{1}{3}$$

$$= \frac{13}{8}$$

$$= 1\frac{5''}{8}$$

The width of each step is $1\frac{5}{8}$ inches.

Second Problem

Width of both A's $= 7\frac{3}{8} - 4\frac{7}{8}$

$= 6\frac{11}{8} - 4\frac{7}{8}$

$= 2\frac{4}{8} = 2\frac{1''}{2}$

Width A (half of above)

$= 2\frac{1}{2} \div 2$

$= \frac{5}{2} \div \frac{2}{1}$

$= \frac{5}{2} \times \frac{1}{2}$

$= \frac{5}{4}$

$= 1\frac{1''}{4}$

The width of A is $1\frac{1}{4}$ inches.

Page 8

(a) $\frac{9}{8}$ feet + $\frac{7}{8}$ feet = $\frac{16}{8}$ feet = 2 feet

(b) $7 \times \frac{5}{16}$ inches $= \frac{7}{1} \times \frac{5}{16} = \frac{35}{16} = 2\frac{3}{16}$ inches.

(c) 20 amperes − 7 amperes = 13 amperes.

(d) \$4000 ÷ \$400 = 10

(e) 32 r.p.m. ÷ 4 = 8 r.p.m.

Page 9

2. (a) **Addition** Overall length = 6'4'' + 6'' = 6' + (4'' + 6'')
= 6 '10'' (6 feet 10 inches)

(b) **Subtraction** Remaining length = 6'6'' − 2'2'' = (6' − 2') + (6'' − 2'')
= 4'4''

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(c) **Multiplication** Length = 3 × 1'3'' = (3 × 1') + (3 × 3'')
= 3'9''

Final Statement: The length of the board is 3 feet 9 inches.

Page 12

(a) The number of pieces cut off is $144 \div 3\frac{1}{6}$

$$= 144 \div \frac{49}{16}$$

$$= \frac{144}{1} \times \frac{16}{49} = \frac{2304}{49} = 47\frac{1}{49}$$

(b) The length of the remaining stock is $\frac{1}{49}$ of $3\frac{1}{16}$

$$= \frac{1}{49} \times \frac{49}{16} = \frac{1}{16}$$

Check: $\frac{1}{48} \times 3' = \frac{3}{48} = \frac{1}{16}$

Page 13

(a) Measure of x $= 6 \times \frac{3''}{16} = \frac{6}{1} \times \frac{3''}{16} = \frac{18}{16} = 1\frac{2''}{16}$ or $1\frac{1''}{8}$

Measure of y $= \frac{1''}{2} + \frac{3''}{16} = \frac{8''}{16} + \frac{3''}{16} = \frac{11''}{16}$

The measure of z $= 1\frac{3''}{4}$

The measure represented by x, y, and z are $1\frac{1''}{8}$, $\frac{11''}{16}$ and $1\frac{3''}{4}$ respectively.

(b) Overall length $= 1\frac{1}{8} + \frac{11}{16} + 1\frac{3}{4} = 1\frac{2}{16} + \frac{11}{16} + 1\frac{12}{16}$

$= 2\frac{25}{16} = 3\frac{9''}{16}$

The overall length of the piston is $3\frac{9''}{16}$.

Check: (Round the figures you obtained in 'a' to the nearer inch.

Length = 1 + 1 + 2
= 4 inches (Compare estimated answer with the actual answer to see if your answer is reasonable.)

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1.
8662
7198
4146
+9123
29129 board feet

2. 1 gallon $- 5\frac{1}{3}$ pts.

8 pts. $- 5\frac{1}{3}$ pts.

$8 - 5\frac{1}{3}$

$7\frac{3}{3} - 5\frac{1}{3} = 2\frac{2}{3}$

$2\frac{2}{3}$ pints remain.

3. Total area to be painted = 20 × 1200 square feet.
= 24 000 square feet

Number of 1000 square feet in total area = 24 000 ÷ 1000
= 24

Cost would be \$45 × 24
= \$1080

$$\begin{array}{r} 45 \\ \times 24 \\ \hline 180 \\ 900 \\ \hline 1080 \end{array}$$

4. $6 = 2 \times 3$
$8 = 2 \times 2 \times 2$
$10 = 2 \times 5$

} break down all numbers into their lowest factors.

LCM multiple of above is: $2 \times 2 \times 2 \times 3 \times 5$ or 120.

The shortest angle iron would be 120 inches or 10 feet long.

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5. The height of the book-case above the lowest shelf to the top of the bookshelft is 64'' − 3'' or 61 inches.

From the 61 inches we must subtract the thickness of 4 shelves (the top shelf and the 3 below it but **not** the bottom shelf.)

So the spaces between the shelves would be

$$\left[61 - \left(4 \times \frac{7}{8}\right)\right] \div 4$$

height total thickness of 4 shelves number of spaces between the shelves

$$= \left[61 - \frac{28}{8}\right] \div 4$$

$$= \left[61 - 3\frac{4}{8}\right] \div 4$$

$$= \left[60\frac{8}{8} - 3\frac{4}{8}\right] \div 4$$

$$= 57\frac{4}{8} \div 4 = 57\frac{1}{2} \div 4 = \frac{115}{2} \times \frac{1}{4} = \frac{115}{8}$$

$$= 14\frac{3}{8} \text{ inches.}$$

The distance between the shelves is $14\frac{3}{8}$ inches.

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6. $A = 1\frac{1}{2} + 2 + 33 + 2 + 1\frac{1}{2} = 40$ inches

   $B = 2$ feet 6 inches $- 2$ feet $4\frac{3}{4} = 1\frac{1}{4}$ inches.

7. Weight of material removed or $52\frac{3}{4} - 35\frac{1}{2}$

   $= 52\frac{3}{4} - 35\frac{2}{4}$

   $= 17\frac{1}{4}$ pounds.

8. To cut the rod into 16 pieces would require only 15 cuts.

   For 15 cuts the amount of waste would be

   $15 \times \frac{1}{15} = \frac{15}{15} = 1$ inch.

   Therefore the length of each piece would be $\frac{44'' - 1''}{16}$.

   $= \frac{43''}{16} = 2\frac{11}{16}$ inches long.

   [3'8'' = 44'']

9. Wall thickness is $\frac{1}{2}\left(9'6'' - 7'4\frac{3''}{4}\right)$

   $= \frac{1}{2}\left(2'1\frac{1''}{4}\right)$

   $= 1'\frac{5''}{8}$ (or $12\frac{5}{8}$ inches thick.)

   OR Alternate solution – first change the measurements above into inches.

10. A = 24'8'' less 8'' = 24'8'' − 8'' = 24' (24 feet)

$B = \frac{1}{4} \times 24' = 6'$ (6 feet)

C = 12'4'' − 4'' = 12'

Total distance would be: C + 24'8'' + C
= 12' + 24'8'' + 12'
= 48'8'' (48 feet 8 inches).

alternative method = 12'4'' + A + 12'4''
= 12'4'' + 24' + 12'4''
= 48'8''

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11. Cubic Yards $= 38\frac{1}{2} \times 7\frac{3}{4}$

$$\frac{77}{2} \times \frac{31}{4} = \frac{2387}{8} = 298\frac{3}{8} \text{ cubic yards.}$$

12. Number of turns $= 3'' \div \frac{1}{64}$

$$= 3 \times \frac{64}{1} = \frac{192}{1} = 192 \text{ turns.}$$

There would be 192 turns necessary.

13. Total Resistance $= 3\frac{1}{2}$ ohms $+ 4\frac{2}{3}$ ohms $+ 7\frac{3}{10}$ ohms.

$$= \left(3\frac{15}{30} + 4\frac{20}{30} + 7\frac{9}{30}\right) \text{ ohms.}$$

$$= 14\frac{44}{30} = 15\frac{14}{30} = 15\frac{7}{15} \text{ ohms.}$$

14. Bending allowance required for each turn $= \frac{1}{2} \times \frac{1''}{8} = \frac{1''}{16}$

Bending allowance required for all turns $= 4 \times \frac{1}{16} = \frac{4}{16} = \frac{1''}{4}$

Total length is:

$3\frac{5}{6} + 1\frac{5}{8} + 5\frac{1}{8} + 1\frac{5}{8} + 1\frac{3}{16} + \frac{1}{4}$ Bending Allowance.

$= 3\frac{5}{16} + 1\frac{10}{16} + 5\frac{2}{16} + 1\frac{10}{16} + 1\frac{3}{16} + \frac{4}{16}$

$= 11\frac{34}{16} = 11 + 2\frac{2}{16} = 11 + 2\frac{1}{8}$

$= 13\frac{1''}{8}$

So $13\frac{1''}{8}$ of strap iron is required.

N.L.C. - B.N.C.
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